## Verification

Please write the names of all group members on the solutions you hand in.

## Problem 1: Invariance Diagrams

Consider the transition system Deque in Figure 1, representing a ring buffer for a double-ended queue. The buffer consists of five cells (represented by integer variables), which can be either free (0) or occupied (1). Starting with a single occupied cell $x_{1}$, we can toggle a cell's state if the states of its neighbors differ.

| $\Theta$ | $x_{1}=1 \wedge x_{2}=0 \wedge x_{3}=0 \wedge x_{4}=0 \wedge x_{5}=0$ |
| :---: | :---: |
| $\rho_{1}$ | $x_{5}+x_{2}=1 \wedge x_{1}^{\prime}=1-x_{1} \wedge \operatorname{pres}\left(x_{2}, x_{3}, x_{4}, x_{5}\right)$ |
| $\rho_{2}$ | $x_{1}+x_{3}=1 \wedge x_{2}^{\prime}=1-x_{2} \wedge \operatorname{pres}\left(x_{1}, x_{3}, x_{4}, x_{5}\right)$ |
| $\rho_{3}$ | $x_{2}+x_{4}=1 \wedge x_{3}^{\prime}=1-x_{3} \wedge \operatorname{pres}\left(x_{1}, x_{2}, x_{4}, x_{5}\right)$ |
| $\rho_{4}$ | $x_{3}+x_{5}=1 \wedge x_{4}^{\prime}=1-x_{4} \wedge \operatorname{pres}\left(x_{1}, x_{2}, x_{3}, x_{5}\right)$ |
| $\rho_{5}$ | $x_{4}+x_{1}=1 \wedge x_{5}^{\prime}=1-x_{5} \wedge \operatorname{pres}\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ |



Figure 1: DEQUE transition system.
Create an invariance diagram which proves for the Deque system that the state with all cells occupied is not reachable.

Hints:

- Keep it simple - the verification diagram in the sample solution only has five nodes.
- State any auxiliary invariants needed to prove P-validity.
- You do not need to give proofs for individual verification conditions.

