# Verification

Lecture 17

Bernd Finkbeiner Peter Faymonville Michael Gerke



# **REVIEW: Timed automaton**

A timed automaton is a tuple

$$TA = (Loc, Act, C, \rightsquigarrow, Loc_0, inv, AP, L)$$
 where:

- Loc is a finite set of locations.
- $Loc_0 \subseteq Loc$  is a set of initial locations
- C is a finite set of clocks
- $L: Loc \rightarrow 2^{AP}$  is a labeling function for the locations
- $\Rightarrow \subseteq Loc \times CC(C) \times Act \times 2^C \times Loc$  is a transition relation, and
- $inv : Loc \rightarrow CC(C)$  is an invariant-assignment function

# **REVIEW: Clock constraints**

Clock constraints over set C of clocks are defined by:

 $g ::= \text{ true } \left| x < c \right| x - y < c \left| x \le c \right| x - y \le c \left| \neg g \right| g \land g$ 

- where  $c \in \mathbb{N}$  and clocks  $x, y \in C$
- rational constants would do; neither reals nor addition of clocks!
- let CC(C) denote the set of clock constraints over C
- ▶ shorthands:  $x \ge c$  denotes  $\neg (x < c)$  and  $x \in [c_1, c_2)$  or  $c_1 \le x < c_2$  denotes  $\neg (x < c_1) \land (x < c_2)$
- Atomic clock constraints do not contain true, ¬ and ∧
  - let ACC(C) denote the set of atomic clock constraints over C
- Simplification: In the following, we assume constraints are diagonal-free, i.e., do neither contain x − y ≤ c nor x − y < c.</p>

# **REVIEW: Guards versus location invariants**





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# Arbitrary clock differences



time --->

#### Composing timed automata

Let  $TA_i = (Loc_i, Act_i, C_i, \rightsquigarrow_i, Loc_{0,i}, inv_i, AP, L_i)$  and H an action-set  $TA_1 \parallel_H TA_2 = (Loc, Act_1 \cup Act_2, C, \rightsquigarrow, Loc_0, inv, AP, L)$  where:

- $Loc = Loc_1 \times Loc_2$  and  $Loc_0 = Loc_{0,1} \times Loc_{0,2}$  and  $C = C_1 \cup C_2$
- $inv(\langle \ell_1, \ell_2 \rangle) = inv_1(\ell_1) \land inv_2(\ell_2)$  and  $L(\langle \ell_1, \ell_2 \rangle) = L_1(\ell_1) \cup L_2(\ell_2)$
- ▶ ~> is defined by the inference rules:

for 
$$\alpha \in H$$
 
$$\frac{\ell_1 \stackrel{g_1:\alpha,D_1}{\sim} \ell'_1 \wedge \ell_2 \stackrel{g_2:\alpha,D_2}{\sim} \ell'_2}{\langle \ell_1, \ell_2 \rangle \stackrel{g_1 \wedge g_2:\alpha,D_1 \cup D_2}{\sim} \langle \ell'_1, \ell'_2 \rangle}$$

for 
$$\alpha \notin H$$
:  $\frac{\ell_1 \overset{g:\alpha,D}{\sim_1} \ell'_1}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha,D}{\sim} \langle \ell'_1, \ell_2 \rangle}$  and  $\frac{\ell_2 \overset{g:\alpha,D}{\sim_2} \ell'_2}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha,D}{\sim} \langle \ell_1, \ell'_2 \rangle}$ 

# **Clock valuations**

- A <u>clock valuation</u> v for set C of clocks is a function  $v : C \longrightarrow \mathbb{R}_{\geq 0}$ 
  - ▶ assigning to each clock  $x \in C$  its current value v(x)
- Clock valuation v+d for  $d \in \mathbb{R}_{\geq 0}$  is defined by:
  - (v+d)(x) = v(x) + d for all clocks  $x \in C$
- Clock valuation reset x in v for clock x is defined by:

$$(\operatorname{reset} x \operatorname{in} v)(y) = \begin{cases} v(y) & \text{if } y \neq x \\ 0 & \text{if } y = x. \end{cases}$$

reset x in (reset y in v) is abbreviated by reset x, y in v

# Timed automaton semantics

For timed automaton  $TA = (Loc, Act, C, \rightsquigarrow, Loc_0, inv, AP, L)$ : Transition system  $TS(TA) = (S, Act', \rightarrow, I, AP', L')$  where:

- $S = Loc \times val(C)$ , state  $s = \langle \ell, v \rangle$  for location  $\ell$  and clock valuation v
- $Act' = Act \cup \mathbb{R}_{\geq 0}$ , (discrete) actions and time passage actions
- ►  $I = \{ \langle \ell_0, v_0 \rangle \mid \ell_0 \in Loc_0 \land v_0(x) = 0 \text{ for all } x \in C \}$
- $AP' = AP \cup ACC(C)$
- ►  $L'(\langle \ell, v \rangle) = L(\ell) \cup \{g \in ACC(C) \mid v \vDash g\}$
- $\blacktriangleright$   $\rightarrow$  is the transition relation defined on the next slide

# Timed automaton semantics

The transition relation  $\rightarrow$  is defined by the following two rules:

- Discrete transition:  $\langle \ell, v \rangle \xrightarrow{d} \langle \ell', v' \rangle$  if all following conditions hold:
  - there is an edge labeled  $(g : \alpha, D)$  from location  $\ell$  to  $\ell'$  such that:
  - g is satisfied by v, i.e.,  $v \models g$
  - v' = v with all clocks in *D* reset to 0, i.e., v' = reset D in v
  - v' fulfills the invariant of location  $\ell'$ , i.e.,  $v' \models inv(\ell')$
- **Delay** transition:  $\langle \ell, v \rangle \xrightarrow{\alpha} \langle \ell, v+d \rangle$  for positive real d
  - if for any  $0 \le d' \le d$  the invariant of  $\ell$  holds for v+d', i.e.  $v+d' \models inv(\ell)$

# Time divergence

- Let for any t < d, for fixed  $d \in \mathbb{R}_{>0}$ , clock valuation  $\eta + t \models inv(\ell)$
- A possible execution fragment starting from the location  $\ell$  is:

$$\langle \ell, \eta \rangle \xrightarrow{d_1} \langle \ell, \eta + d_1 \rangle \xrightarrow{d_2} \langle \ell, \eta + d_1 + d_2 \rangle \xrightarrow{d_3} \langle \ell, \eta + d_1 + d_2 + d_3 \rangle \xrightarrow{d_4} \dots$$

- where  $d_i > 0$  and the infinite sequence  $d_1 + d_2 + ...$  converges towards d
- such path fragments are called time-convergent
- ⇒ time advances only up to a certain value
- Time-convergent execution fragments are unrealistic and ignored
  - much like unfair paths (as we will see later on)

# Time divergence

- Infinite path fragment  $\pi$  is <u>time-divergent</u> if *ExecTime*( $\pi$ ) =  $\infty$
- The function *ExecTime* :  $Act \cup \mathbb{R}_{>0} \rightarrow \mathbb{R}_{\geq 0}$  is defined as:

$$ExecTime(\tau) = \begin{cases} 0 & \text{if } \tau \in Act \\ d & \text{if } \tau = d \in \mathbb{R}_{>0} \end{cases}$$

• For infinite execution fragment  $\rho = s_0 \xrightarrow{\tau_1} s_1 \xrightarrow{\tau_2} s_2 \dots$  in TS(TA) let:

ExecTime
$$(\rho) = \sum_{i=0}^{\infty} ExecTime(\tau_i)$$

- for path fragment π in TS(TA) induced by ρ:
  ExecTime(π) = ExecTime(ρ)
- For state *s* in *TS*(*TA*):

 $Paths_{div}(s) = \{ \pi \in Paths(s) \mid \pi \text{ is time-divergent } \}$ 

# Example: light switch



The path  $\pi$  in *TS*(*Switch*) in which on- and of-periods of one minute alternate:

 $\pi = \langle off, 0 \rangle \langle off, 1 \rangle \langle on, 0 \rangle \langle on, 1 \rangle \langle off, 1 \rangle \langle off, 2 \rangle \langle on, 0 \rangle \langle on, 1 \rangle \langle off, 1 \rangle \dots$ 

is <u>time-divergent</u> as *ExecTime*( $\pi$ ) = 1 + 1 + 1 + ... =  $\infty$ . The path:

$$\pi' = \langle off, 0 \rangle \langle off, 1/2 \rangle \langle off, 3/4 \rangle \langle off, 7/8 \rangle \langle off, 15/16 \rangle \dots$$

is <u>time-convergent</u>, since *ExecTime*( $\pi'$ ) =  $\sum_{i \ge 1} \left(\frac{1}{2}\right)^i = 1 < \infty$ 

# Timelock

- State  $s \in TS(TA)$  contains a <u>timelock</u> if  $Paths_{div}(s) = \emptyset$ 
  - there is no behavior in s where time can progress ad infinitum
  - clearly: any terminal state contains a timelock (but also non-terminal states may do)
  - terminal location does not necessarily yield a state with timelock (e.g. inv = true)
- TA is <u>timelock-free</u> if no state in Reach(TS(TA)) contains a timelock
- Timelocks are considered as modeling flaws that should be avoided

#### Zenoness

- A TA that performs infinitely many actions in finite time is Zeno
- Path  $\pi$  in *TS*(*TA*) is <u>Zeno</u> if:
  - it is time-convergent, and
  - ► infinitely many actions  $\alpha \in Act$  are executed along  $\pi$
- TA is <u>non-Zeno</u> if there does not exist an initial Zeno path in TS(TA)
  - any  $\pi$  in TS(TA) is time-divergent or
  - is time-convergent with nearly all (i.e., all except for finitely many) transitions being delay transitions
- Zeno paths are considered as modeling flaws that should be avoided

# A sufficient criterion for Non-Zenoness

Let *TA* with set *C* of clocks such that for every control cycle:

$$\ell_0 \overset{g_1:\alpha_1,C_1}{\rightsquigarrow} \ell_1 \overset{g_2:\alpha_2,C_2}{\rightsquigarrow} \dots \overset{g_n:\alpha_n,C_n}{\rightsquigarrow} \ell_n$$

there exists a clock  $x \in C$  such that:

- 1.  $x \in C_i$  for some  $0 < i \le n$ , and
- 2. there exists a constant  $c \in \mathbb{N}_{>0}$  such that for all clock evaluations  $\eta$ :

 $\eta(x) < c$  implies ( $\eta \neq g_j$  or  $\eta \neq inv(\ell_j)$ ), for some  $0 < j \le n$ 

Then: TA is non-Zeno

# Timelock, time-divergence and Zenoness

 A timed automaton is only considered an adequate model of a time-critical system if it is:

non-Zeno and timelock-free

 Time-convergent paths will be explicitly excluded from the analysis.

# Timed CTL

Syntax of TCTL <u>state-formulas</u> over *AP* and set *C*:

$$\Phi ::= \mathsf{true} \left| \begin{array}{c} a \end{array} \right| \left| \begin{array}{c} g \end{array} \right| \left| \begin{array}{c} \Phi \end{array} \wedge \left| \begin{array}{c} \Phi \end{array} \right| \left| \begin{array}{c} \neg \Phi \end{array} \right| \left| \begin{array}{c} \mathsf{E} \varphi \end{array} \right| \left| \begin{array}{c} \mathsf{A} \varphi \end{array} \right|$$

where  $a \in AP$ ,  $g \in ACC(C)$  and  $\varphi$  is a path-formula defined by:

$$\varphi ::= \Phi U^{J} \Phi$$

where  $J \subseteq \mathbb{R}_{\geq 0}$  is an interval whose bounds are naturals Forms of J: [n, m], (n, m], [n, m) or (n, m) for  $n, m \in \mathbb{N}$  and  $n \leq m$ 

for right-open intervals,  $m = \infty$  is also allowed

#### Some abbreviations

 $\diamond \diamond^{J} \Phi = \text{true } \mathsf{U}^{J} \Phi$  $\diamond \mathsf{E} \Box^{J} \Phi = \neg \mathsf{A} \diamond^{J} \neg \Phi \text{ and } \mathsf{A} \Box^{J} \Phi = \neg \mathsf{E} \diamond^{J} \neg \Phi$  $\diamond \Phi = \diamond^{[0,\infty)} \Phi \text{ and } \Box \Phi = \Box^{[0,\infty)} \Phi$ 

# Semantics of TCTL

For state  $s = \langle \ell, \eta \rangle$  in *TS*(*TA*) the satisfaction relation  $\vDash$  is defined by:

s ⊨ true		
$s \vDash a$	iff	$a \in L(\ell)$
$s \vDash g$	iff	$\eta \vDash g$
$S\vDash \neg  \Phi$	iff	not $s \models \Phi$
$\mathbf{S} \vDash \Phi \land \Psi$	iff	$(s \models \Phi)$ and $(s \models \Psi)$
$s \vDash E \varphi$	iff	$\pi \vDash \varphi$ for some $\pi \in Paths_{div}(s)$
$s \vDash A \varphi$	iff	$\pi \vDash \varphi$ for all $\pi \in Paths_{div}(s)$

path quantification over time-divergent paths only

# The $\Rightarrow$ relation

For infinite path fragments in TS(TA) performing  $\infty$  many actions let:

 $s_0 \xrightarrow{d_0} s_1 \xrightarrow{d_1} s_2 \xrightarrow{d_2} \dots$  with  $d_0, d_1, d_2 \dots \ge 0$ 

denote the equivalence class containing all infinite path fragments induced by execution fragments of the form:



where  $k_i \in \mathbb{N}$ ,  $d_i \in \mathbb{R}_{\geq 0}$  and  $\alpha_i \in Act$  such that  $\sum_{j=1}^{k_i} d_i^j = d_i$ . Notation:  $s_i + d = \langle \ell_i, \eta_i + d \rangle$  where  $s_i = \langle \ell_i, \eta_i \rangle$ .

# Semantics of TCTL

For time-divergent path  $\pi \in s_0 \xrightarrow{d_0} s_1 \xrightarrow{d_1} \ldots$ :

 $\pi \vDash \Phi \, \mathsf{U}^{\mathsf{J}} \, \Psi$ 

#### iff

 $\exists i \ge 0. s_i + d \models \Psi \text{ for some } d \in [0, d_i] \text{ with } \sum_{k=0}^{i-1} d_k + d \in J$ and  $\forall j \le i. s_j + d' \models \Phi \lor \Psi \text{ for every } d' \in [0, d_j] \text{ with } \sum_{i=0}^{j-1} d_k + d' \le \sum_{k=0}^{i-1} d_k + d$ 

# TCTL-semantics for timed automata

- Let *TA* be a timed automaton with clocks *C* and locations *Loc*
- For TCTL-state-formula Φ, the satisfaction set Sat(Φ) is defined by:

$$Sat(\Phi) = \{ s \in Loc \times Eval(C) \mid s \models \Phi \}$$

• TA satisfies TCTL-formula  $\Phi$  iff  $\Phi$  holds in all initial states of TA:

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TA \models \Phi if and only if \forall \ell_0 \in Loc_0. \langle \ell_0, \eta_0 \rangle \models \Phi
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where  $\eta_0(x) = 0$  for all  $x \in C$ 

# Timed CTL versus CTL

Due to ignoring time-convergent paths in TCTL semantics, possibly:

$$\underbrace{TS(TA) \vDash_{\mathsf{TCTL}} \mathsf{A} \varphi}_{\mathsf{TCTL semantics}} \quad \mathsf{but} \quad \underbrace{TS(TA) \not\models_{\mathsf{CTL}} \mathsf{A} \varphi}_{\mathsf{CTL semantics}}$$

- CTL semantics considers all paths, timed CTL only time-divergent paths
- For  $\Phi = A \Box (on \longrightarrow A \Diamond off)$  and the light switch

 $TS(Switch) \vDash_{TCTL} \Phi$  whereas  $TS(TA) \not\models_{CTL} \Phi$ 

 there are time-convergent paths on which location on is never left

# Characterizing timelock

- TCTL semantics is also well-defined for TA with timelock
- A state is <u>timelock-free</u> if and only if it satisfies E □ true
  - some time-divergent path satisfies □true, i.e., there is ≥ 1 time-divergent path
  - note: for fair CTL, the states in which a fair path starts also satisfy E □ true
- ► *TA* is timelock-free iff  $\forall s \in Reach(TS(TA))$ :  $s \models E \square$  true
- Timelocks can thus be checked by model checking

# TCTL model checking

• TCTL model-checking problem:  $TA \models \Phi$  for non-Zeno TA

$TA \vDash \Phi$	iff	$TS(TA) \vDash \Phi$
$\smile$		$\frown$
timed automaton		infinite state graph

- Idea: consider a finite region graph RG(TA)
- Transform TCTL formula  $\Phi$  into an "equivalent" CTL-formula  $\widehat{\Phi}$
- Then:  $TA \vDash_{\mathsf{TCTL}} \Phi$  iff  $RG(TA) \vDash_{\mathsf{CTL}} \widehat{\Phi}$

finite state graph

# Eliminating timing parameters

- Eliminate all intervals  $J \neq [0, \infty)$  from TCTL formulas
  - introduce a fresh clock, z say, that does not occur in TA
  - $s \models \mathsf{E} \diamondsuit^{\mathsf{J}} \Phi$  iff reset z in  $s \models \diamondsuit(z \in \mathsf{J} \land \Phi)$
- Formally: for any state s of TS(TA) it holds:

$$s \vDash \mathsf{E} \Phi \mathsf{U}^{\mathsf{J}} \Psi$$
 iff  $\underbrace{s\{z := 0\}}_{\text{state in } TS(TA \oplus z)} \vDash \mathsf{E} \mathsf{E} ((\Phi \lor \Psi) \mathsf{U} (z \in \mathsf{J}) \land \Psi)$ 

$$s \models A \Phi U^{J} \Psi$$
 iff  $\underbrace{s\{z := 0\}}_{\text{state in } TS(TA \oplus z)} \models A((\Phi \lor \Psi) U(z \in J) \land \Psi)$ 

• where  $TA \oplus z$  is TA (over C) extended with  $z \notin C$ 

# **Clock equivalence**

Impose an equivalence, denoted  $\cong$ , on the clock valuations such that:

(A) Equivalent clock valuations satisfy the same clock constraints g in *TA* and  $\Phi$ :

$$\eta \cong \eta' \Rightarrow \begin{pmatrix} \eta \vDash g & \text{iff} & \eta' \vDash g \end{pmatrix}$$

- no diagonal clock constraints are considered
- all the constraints in TA and Φ are thus either of the form x ≤ c or x < c</p>
- (B) Time-divergent paths emanating from equivalent states are equivalent
  - this property guarantees that equivalent states satisfy the same path formulas
- (C) The number of equivalence classes under  $\cong$  is finite

# **First observation**

- $\eta \models x < c$  whenever  $\eta(x) < c$ , or equivalently,  $\lfloor \eta(x) \rfloor < c$ 
  - ▶  $\lfloor d \rfloor = \max \{ c \in \mathbb{N} \mid c \leq d \}$  and  $frac(d) = d \lfloor d \rfloor$
- $\eta \models x \le c$  whenever  $\lfloor \eta(x) \rfloor < c$  or  $\lfloor \eta(x) \rfloor = c$  and  $frac(\eta(x)) = 0$
- $\Rightarrow \eta \models g$  only depends on  $\lfloor \eta(x) \rfloor$ , and whether  $frac(\eta(x)) = 0$ 
  - Initial suggestion: clock valuations  $\eta$  and  $\eta'$  are equivalent if:

 $\lfloor \eta(x) \rfloor = \lfloor \eta'(x) \rfloor$  and  $frac(\eta(x)) = 0$  iff  $frac(\eta'(x)) = 0$ 

▶ Note: it is crucial that in *x* < *c* and *x* ≤ *c*, *c* is a natural

# Second observation

- Consider location l with inv(l) = true and only outgoing transitions:
  - one guarded with  $x \ge 2$  (action  $\alpha$ ) and y > 1 (action  $\beta$ )
- Let state  $s = \langle \ell, \eta \rangle$  with  $1 < \eta(x) < 2$  and  $0 < \eta(y) < 1$ 
  - $\alpha$  and  $\beta$  are disabled, only time may elapse
- ► Transition that is enabled next depends on x < y or x ≥ y</p>
  - e.g., if  $frac(\eta(x)) \ge frac(\eta(y))$ , action  $\alpha$  is enabled first
- Suggestion for strengthening of initial proposal for all x, y ∈ C by:

 $frac(\eta(x)) \leq frac(\eta(y))$  if and only if  $frac(\eta'(x)) \leq frac(\eta'(y))$ 

# **Final observation**

- So far, clock equivalence yield a denumerable though not finite quotient
- For  $TA \models \Phi$  only the clock constraints in TA and  $\Phi$  are relevant
  - let  $c_x \in \mathbb{N}$  the <u>largest constant</u> with which x is compared in TA or  $\Phi$
- $\Rightarrow$  If  $\eta(x) > c_x$  then the actual value of x is irrelevant
  - ► constraints on  $\cong$  so far are only relevant for clock values of x(y)up to  $c_x(c_y)$

# **Midterm Review**

# Verification -- Part I

- Transition systems: sequential circuits, concurrent systems, channel systems
- Linear-time properties: safety vs. liveness
- Regular properties: Büchi automata
- LTL: from LTL to Büchi automata, LTL model checking
- CTL\*: LTL vs. CTL, fairness, model checking
- Symbolic verification: BDDs, bounded model checking
- Implementation relations: Bisimulation, simulation, stuttering

 $AXAGp \equiv AGAXp$ 

 $\mathsf{EX} \mathsf{EG} p \equiv \mathsf{EG} \mathsf{EX} p$ 

AF AG *p* can be expressed in LTL.

# If $\Phi$ is a CTL formula and $\psi$ is an LTL formula such that $\Phi \equiv \psi$ , then $\neg \Phi \equiv \neg \psi$ .

 $s \models \mathsf{EF} \mathsf{EG} p$  iff there is a path  $\pi$  from s with  $\pi \models \mathsf{F} \mathsf{G} p$ 

 $s \models EG EF p$  iff there is a path  $\pi$  from s with  $\pi \models GF p$  Let *TS* be a transition system and  $\Phi$  a CTL formula. If *TS* does <u>not</u> satisfy  $\neg \Phi$ , then *TS* satisfies  $\Phi$ .

#### Let $s_1, s_2$ be states of a transition system and let

$$\Phi = \mathsf{E}(a \mathsf{U}(\mathsf{EX} b \land \mathsf{EX} c)).$$

If  $s_1 \models \Phi$  and  $\underline{\text{not}} s_2 \models \Phi$ then  $Traces(s_1) \neq Traces(s_2)$ .

CTL\* equivalence is strictly finer than CTL equivalence.



LTL equivalence is strictly finer than CTL equivalence.



CTL equivalence is strictly finer than LTL equivalence.

If  $s \models AFp$ then  $s \models_{fair} AFp$ 

If  $s \models \mathsf{EF} p$ then  $s \models_{fair} \mathsf{EF} p$ 

 $s \vDash_{fair} E(a \cup b)$  iff  $s \vDash E(a \cup (b \land EG true))$ 

$$s \vDash_{fair} E(a \cup b) \text{ iff}$$
  
 $s \vDash E(a \cup (b \land a_{fair}))$ 

where  $a_{fair}$  is an atomic proposition with  $s \models a_{fair}$  iff  $s \models_{fair}$  EG true

For each Büchi automaton A there is an LTL formula  $\varphi$  such that Words( $\varphi$ ) is the language of A.

If two states  $s_1$  an  $s_2$  in a finite transition system satisfy the same  $CTL_{\U}$  formulas, then  $s_1$  and  $s_2$  are bisimilar.



Bisimilar transition systems are simulation equivalent.

The following two transition systems are stutter-trace equivalent.





#### Let $TS_1$ and $TS_2$ be two stutter-bisimilar transition systems and let $\varphi$ be an LTL formula without Next

then either both  $TS_1$  and  $TS_2$  satisfy  $\varphi$  or neither satisfies  $\varphi$ .

The following two transition systems are divergence-sensitive stutter-bisimilar.



For every boolean function there is a variable ordering such that the size of the ROBDD is polynomial.

For every boolean function there is a variable ordering such that the size of the ROBDD is exponential.