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Summer term 2015
Problem Set 7
Due: Tuesday, June 16th, 2015

Recursion Theory

Problem 1: A "Wrong" Definition — 4 Points

For a given partial function $g\colon \mathbb{N}^{n+1} \to \mathbb{N}$ define the partial function $f\colon \mathbb{N}^n \to \mathbb{N}$ by

$$f(\overline{x}) = \begin{cases} \min\{y \mid g(\overline{x}, y) = 0\} & \text{if } \{y \mid g(\overline{x}, y) = 0\} \neq \emptyset, \\ \bot & \text{otherwise.} \end{cases}$$

We write $f(\overline{x}) = \hat{\mu} y\colon g(\overline{x}, y) = 0$ for short.

Show that there is a μ-recursive function g such that $\hat{\mu} y\colon g(\overline{x}, y) = 0$ is not μ-recursive.

Problem 2: The Function U — 2 Points

Let $f(x) = c_0^{(1)}(\mu y\colon T_1 xxy)$.

Show that there is no primitive recursive relation $R \subseteq \mathbb{N} \times \mathbb{N}$ such that $f(x) = \mu y\colon Rxy$.

This implies that the *output function* U in the statement of the Kleene Normal Form theorem is necessary.

Problem 3: Fun with Functions — 2 + 2 + 2 Points

1. Show that there is an e with $\varphi_e(x) = e^2 + x$.
2. Show that there is an e with $\operatorname{dom}(\varphi_e) = \{e^2 + e\}$.
3. Show that there is total μ-recursive function $t\colon \mathbb{N} \to \mathbb{N}$ such that $\varphi_{t(k)}$ is the function $x \mapsto x^k$ for every k.