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Summer term 2015
Problem Set 3
Due: Tuesday, May 12th, 2015

Recursion Theory

Problem 1: Primitive Recursive Functions 1 + 1 + 1 + 1 + 2 + 2 + 2 + 1 +1 Points

Show that the following functions and predicates are primitive recursive.

a) $x \dot{-} y = \begin{cases} x - y & \text{if } x \geq y, \\ 0 & \text{otherwise.} \end{cases}$

b) The predicate $O = \{1, 3, 5, \ldots\}$ containing the odd numbers,

c) The predicate $<$ over $\mathbb{N}$.

d) $\text{div3}(x) = \lfloor \frac{x}{3} \rfloor$.

e) $\text{rem}(x, y) = x \bmod y$ (convention: $\text{rem}(x, 0) = 0$).

f) $\text{div}(x, y) = \lfloor \frac{x}{y} \rfloor$ (convention: $\text{div}(x, 0) = 0$).

g) Every function $f \colon \mathbb{N} \to \mathbb{N}$ with $f(x + y) = f(x) + f(y)$.

h) The predicate $P = \{n \mid \text{the decimal expansion of } \pi \text{ contains the infix } 0^n\}$.

i) The binary minimum function $\min \colon \mathbb{N}^2 \to \mathbb{N}$.

Hint: Pick a suitable recursion parameter for e) and f).