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Summer term 2015
Problem Set 2
Due: Tuesday, May 5th, 2015

Recursion Theory

Please indicate your **name** and your **matriculation number** on your solution and hand it in before the tutorial. We encourage you to collaborate in **groups** of up to **two** students. Only one submission per group is necessary.

Problem 1: Minimality — 2 + 2 + 2 Points

Show that the instruction set of register machines is minimal in the following sense: for every type of instruction (increment, decrement, and conditional jump), give a function that is computable by a URM, and prove that it is not computable by a URM without this type of instruction.

Problem 2: Encoding URMs — 0.5 + 1 + 0.5 Points

Consider the following URM $\mathcal{P}$:

1. DEC(X_1)
2. IF $X_1 > 0$ GOTO 1
3. STOP

a) Determine for every $n \in \mathbb{N}$ the function $f\colon \mathbb{N}^n \to \mathbb{N}$ computed by $\mathcal{P}$.

b) Determine code($\mathcal{P}$).

c) Determine a number e that does not encode a register machine.

Problem 3: — 2 Points

Show: every URM-computable function $f\colon \mathbb{N} \to \mathbb{N}$ has infinitely many indices, i.e., numbers $e \in \mathbb{N}$ with $\varphi_e = f$.

Problem 4: — 1 + 1 Points

a) Determine $\varphi_e(x)$ for $e = 131249$ and $x = 3$.

b) Determine $\varphi_e(x)$ for $e = 24575$ and $x = 2$.