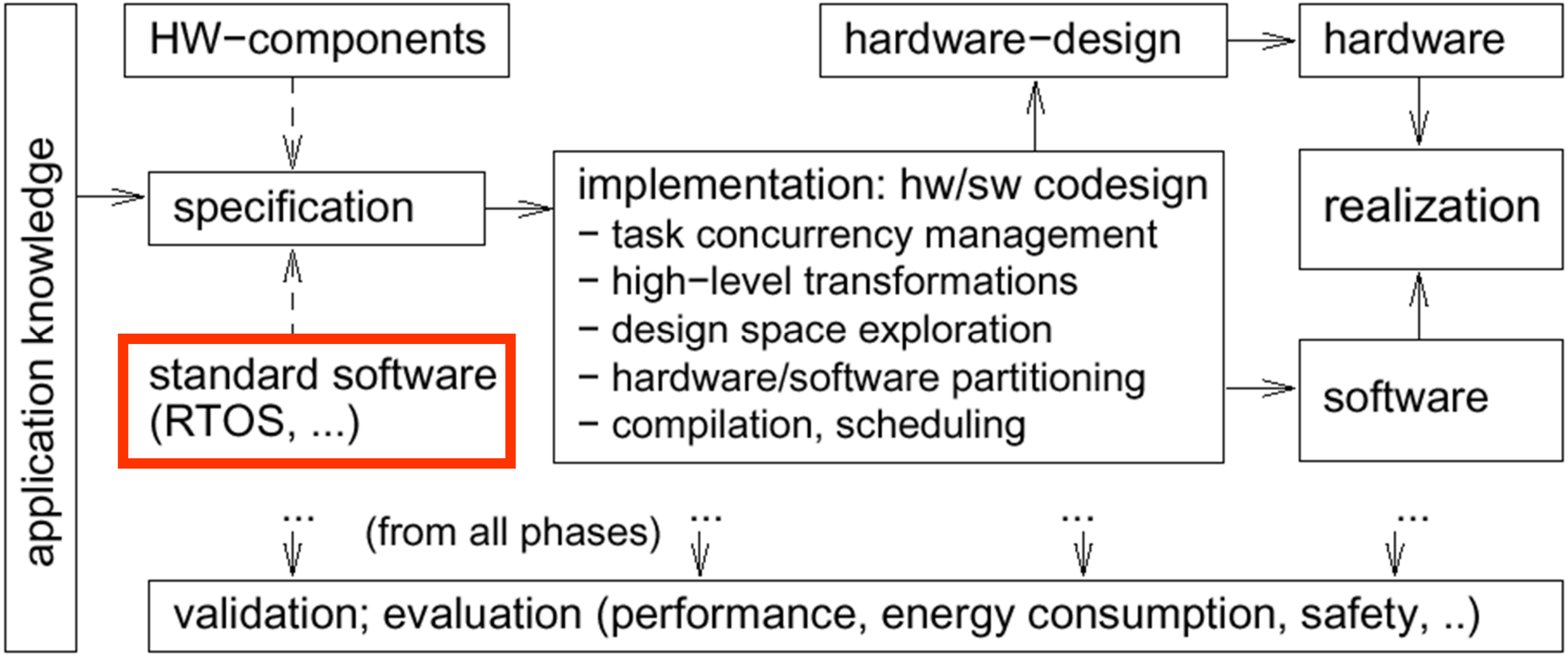
Embedded Systems

Overview of embedded systems design REVIEW



Scheduling

REVIEW

- Support for multi-tasking/multi-threading – several tasks to run on shared resources
- **Task** ~ process – sequential program
- **Resources**: processor(s) + memory, disks, buses, communication channels, etc.
- Scheduler assigns **shared resources** to tasks for **durations of time**
- Most important resource(s) – processor(s)
- **Scheduling** – mostly concerned with processor(s)
  - **Online** – scheduling decisions taken when input becomes available
  - **Offline** – schedule computed with complete input known
- Other shared resources with exclusive access complicate scheduling task

Point of departure: Scheduling general IT systems

REVIEW

- In general IT systems, not much is known about the set of tasks a priori
  - The set of tasks to be scheduled is dynamic:
    - new tasks may be inserted into the running system,
    - executed tasks may disappear.
    - Tasks are activated with unknown activation patterns.
  - The power of schedulers thus is inherently limited by lack of knowledge – only online scheduling is possible

Scheduling processes in ES: The difference in process charaterization

REVIEW

- Most ES are “closed shops”
  - Task set of the system is known
  - at least part of their activation patterns is known
    - periodic activation in, e.g., signal processing
    - maximum activation frequencies of asynchronous events determinable from environment dynamics, minimal inter-arrival times
  - Possible to determine bounds on their execution time (WCET)
    - if they are well-built
    - if we invest enough analysis effort
- Much better prospects for **guaranteeing response times** and for delivering *high-quality* schedules!

Scheduling processes in ES: Differences in goals

REVIEW

- In classical OS, quality of scheduling is normally measured in terms of performance:
  - Throughput, reaction times, … in average case
- In ES, the schedules do often have to meet stringent quality criteria under all possible execution scenarios:
  - A task of an RTOS is usually connected with a **deadline**. Standard operating systems do not deal with deadlines.
    - There are hard deadlines which have to be fulfilled under all circumstances and
    - "soft deadlines" which should be fulfilled if possible
  - Scheduling of an RTOS has to be predictable.
  - Real-time systems have to be designed for peak load. Scheduling for meeting deadlines should work for all anticipated situations.

Constraints for real-time tasks

REVIEW

- Three types of constraints for real-time tasks:
  - Timing constraints
  - Precedence constraints (priority c.)
  - Mutual exclusion constraints on shared resources

REVIEW

Model-based Code Generation: Esterel Scade

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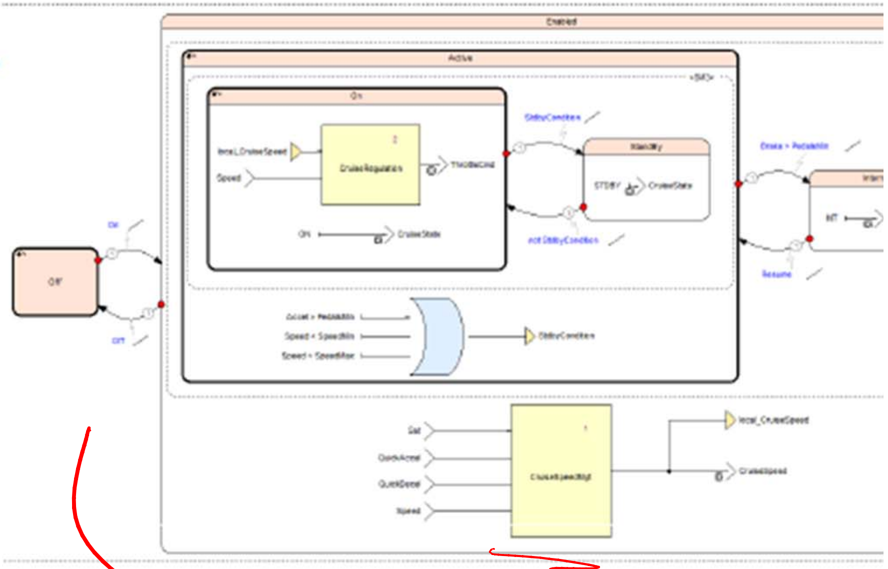
Model-based Software Development

- Model is software specification.

- Hardware/Software codesign.
- Prototyping.
- Formal verification.

- Automated & integrated development tools:
  - Simulation.
  - Documentation.
  - Automatic code generation.

- Automated & integrated verification and test methods
  - Model checking
  - Static system analysis
  - Synthesis of test suites



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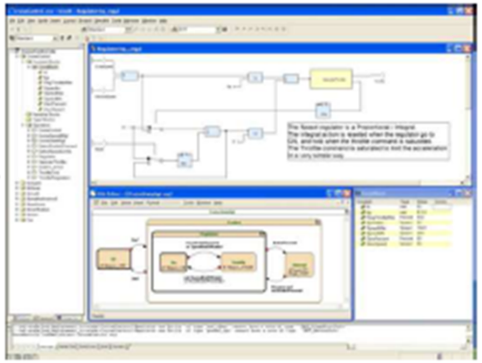
Model-based Software Development



Generator

SCADE programs
(~Esterel/Lustre)

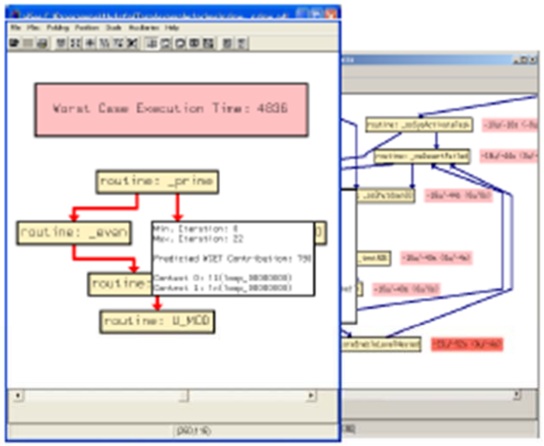
Compiler

C Code

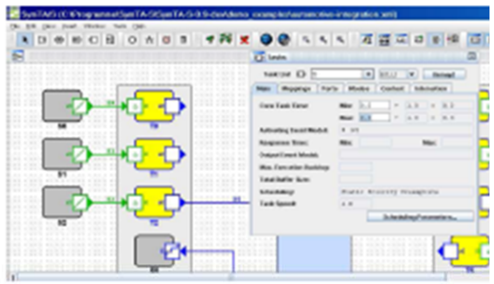
Compiler

Binary Code

Esterel Scade Suite
- SCADE language
- SyncCharts/SSM



- aiT WCET Analyzer
- StackAnalyzer



SymTA/S
- System-level Schedulability Analysis

SyncCharts

REVIE

- Visual formalism for describing states and transitions of a system in a modular fashion.
- Extension of state-transition diagrams (Mealy/Moore automa
  - Hierarchy
  - Modularity
  - Parallelism
- Is fully deterministic.
- Tailored to control-oriented applications (drivers, protocols).

9

- Implements sychronous principle.

Synchronous Programming

REVIEW

- Important requirement: guaranteeing deterministic behavior.
- Time is divided into discrete ticks (also called cycles, steps, instants).
- Simple implementation: sampling / cyclic executive:

```
<Initialize Memory>
Foreach period do
        <Read Inputs>
        <Compute Outputs>
        <Update Memory>
```

- Verification of timing behavior: prove that the worst-case execution time (WCET) of any reaction fits between two iterations of the cyclic executive.
- Implicit assumption: presence of a global clock. This makes application in distributed environments difficult.

Overview

REVIEW

- StateCharts:
  - First, and probably most popular formal language for the design of reactive systems.
  - Focus on specification and design, not designed as a programming language.
  - Determinism is not ensured.
  - No standardized semantics.
- Programming languages for designing reactive systems:
  - ESTEREL [Berry]: textual imperative language.
  - LUSTRE [Caspi, Halbwachs]: textual declarative language. Tailored to data-flow oriented systems (e.g. regulation systems).
  - SCADE [Esterel Inc.]. Enhanced LUSTRE, graphical and textual formalism.
  - SyncCharts / SSM: Graphical formalism corresponding to ESTEREL.

Concurrency vs. Parallism

REVIEW

- Concurrency is central to embedded systems. A computer program is said to be concurrent if different parts of the program conceptually execute simultaneously.

- A program is said to be parallel if different parts of the program physically execute simultaneously on distinct hardware (multi-core, multi-processor or distributed systems)

Petri Nets

REVIEW

Petri's nets - complex foundations for simple models

For his nets, Carl Adam Petri has made an attempt to combine automata from theoretical CS, insights from physics, and pragmatic expertise from engineers:

- *state is distributed, transitions are localised (space is relevant)*
- *local causality replaces global time (time as a derived concept)*
- *subsystems interact by explicit communication*
  (information transport is as relevant as information processing)

engineers can often ignore the background - Petri nets just work!

(but the background explains why things work, why concepts from other disciplines, such as logic, have been integrated into Petri nets so easily, and why foundational research has to continue)

Application areas

REVIEW

- modelling, analysis, verification of distributed systems
- automation engineering
- business processes
- modeling of resources
- modeling of synchronization

Key Elements

REVIEW

- **Conditions**
  Either met or not met. Conditions represent "local states". Set of conditions describes the potential state space.
- **Events**
  May take place if certain conditions are met. Event represents a state transition.
- **Flow relation**
  Relates conditions and events, describes how an event changes the local and global state.
- **Tokens**
  Assignments of tokens to conditions specifies a global state.

Conditions, events and the flow relation form a **bipartite graph** (graph with two kinds of nodes).

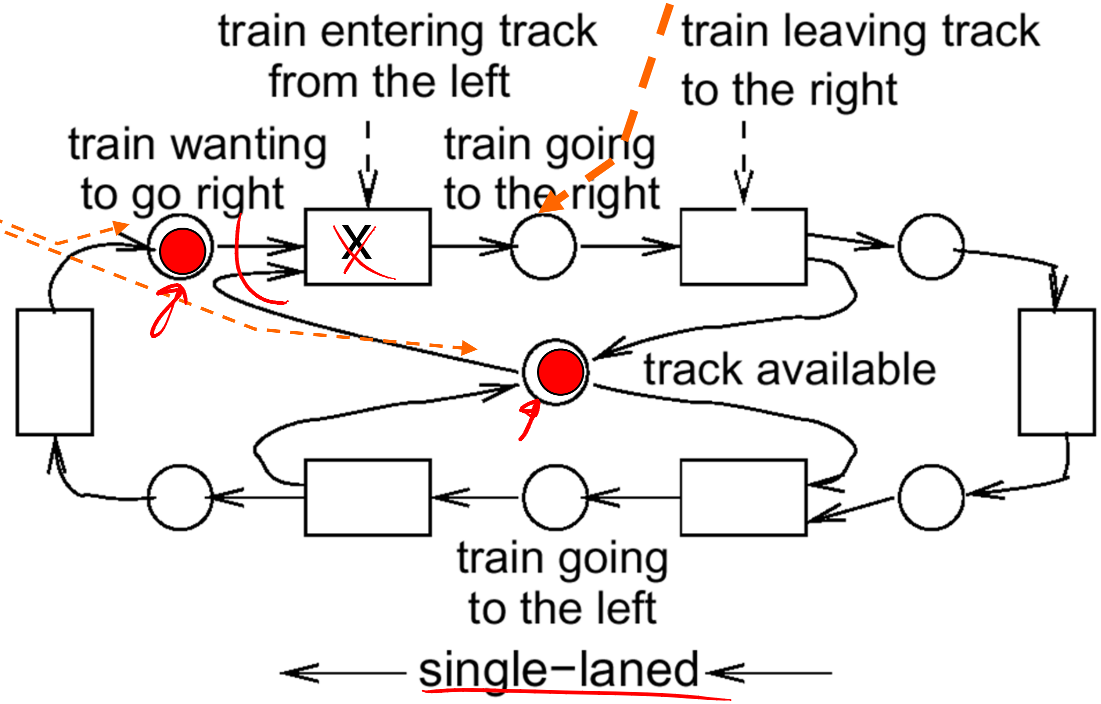
Example 2: Synchronization at single track rail segment

REVIEW

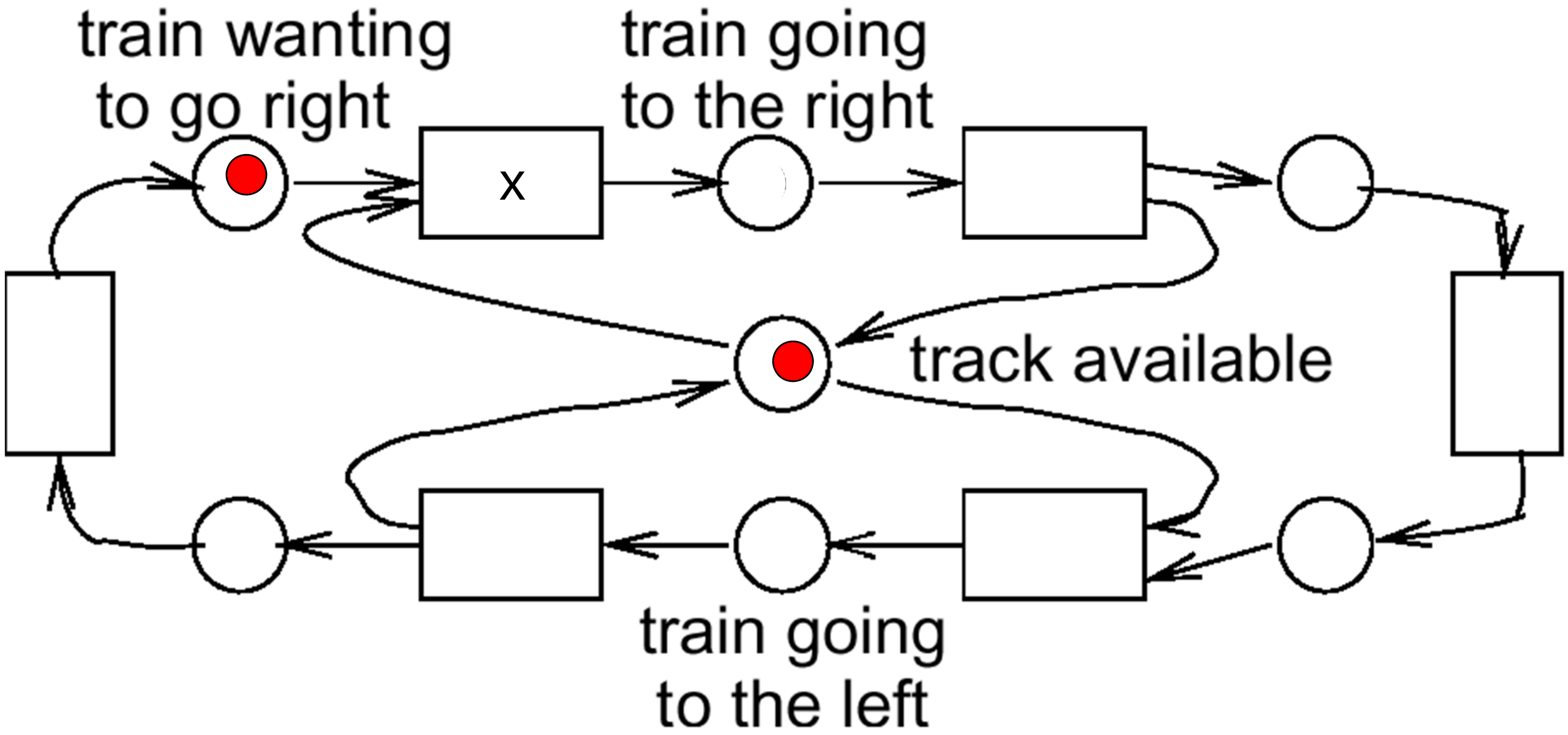
- **mutual exclusion: there is at most one train using the track rail**

„Postcondition“ of x fulfilled

„Preconditions“ of x fulfilled



Playing the „token game“: dynamic behavior



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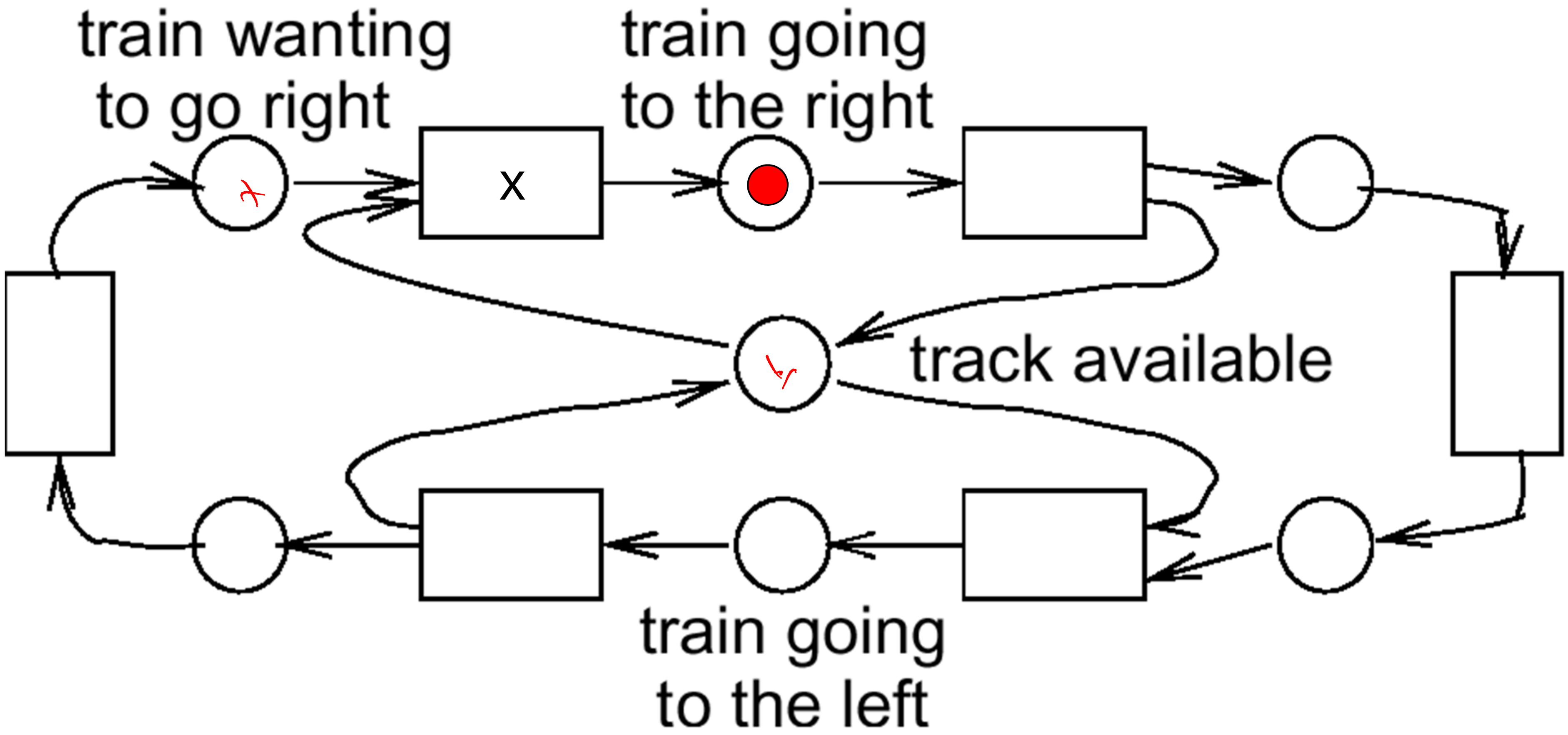
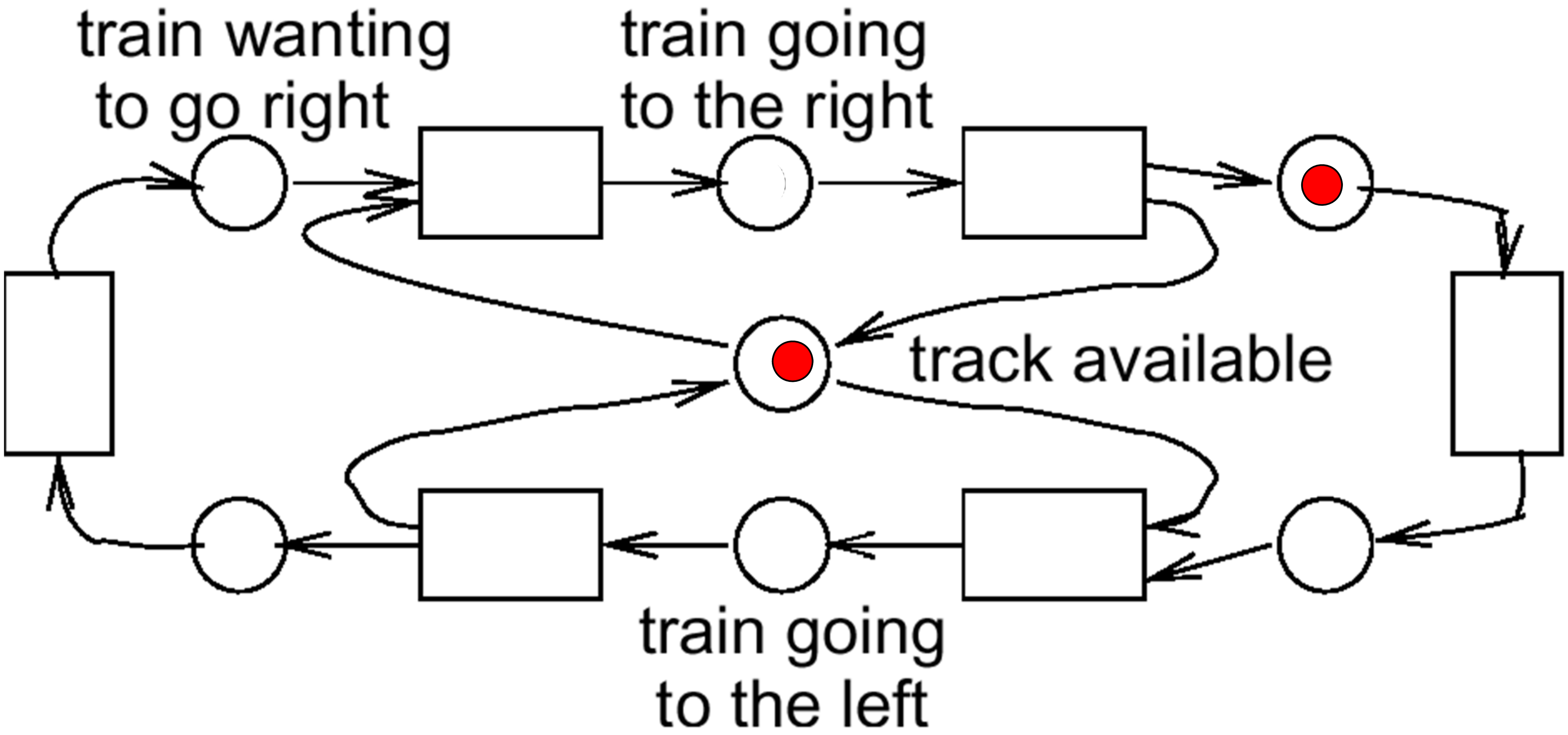


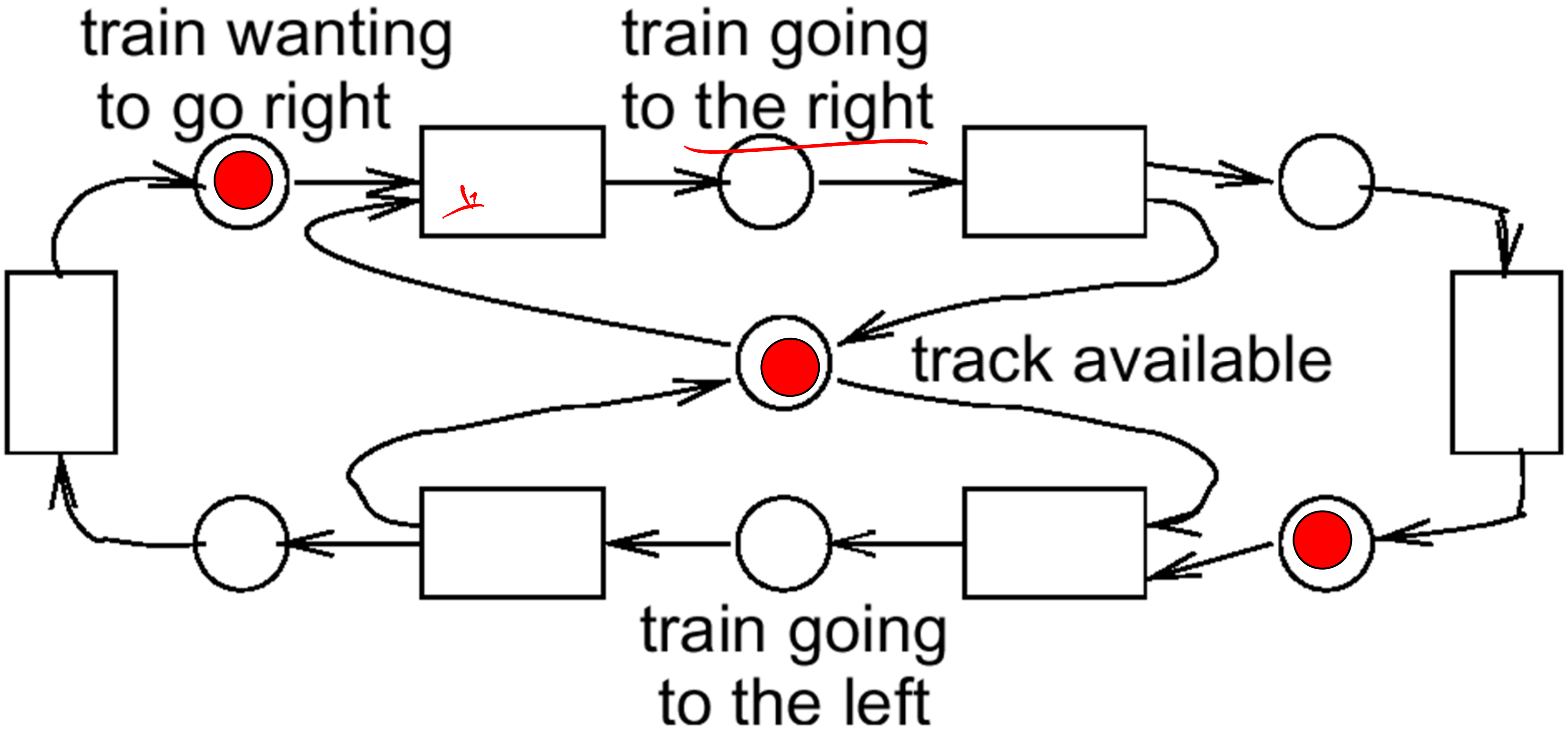
Playing the „token game“: dynamic behavior

REVIEW



Conflict for resource „track“: two trains competing

REVIEW



A Petri nets is nondeterministic

When multiple transitions are enabled at the same time, any one of them may fire.

If a transition is enabled, it may fire (but it doesn't have to).

Condition/event Petri nets

REVIEW

single token per place

Def.: $N=(C,E,F)$ is called a **net**, iff the following holds

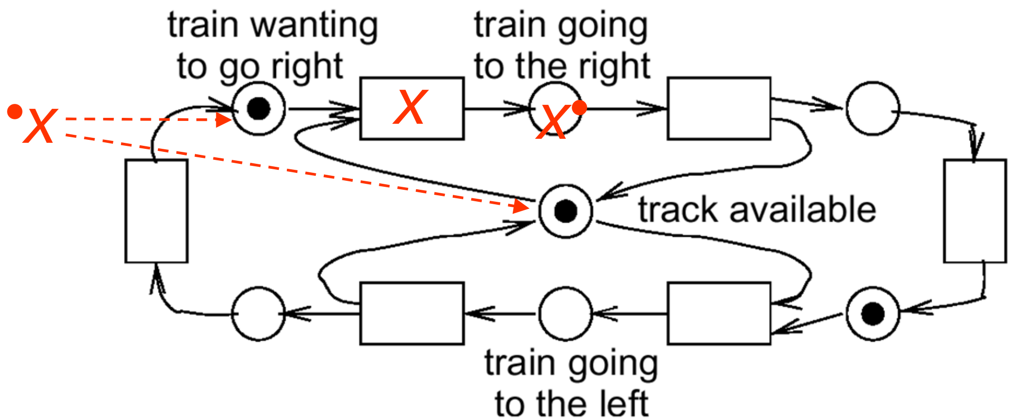
1. C and E are disjoint sets
2. $F \subseteq (C \times E) \cup (E \times C)$; is binary relation, („**flow relation**“)

Def.: Let N be a net and let $x \in (C \cup E)$.

$^{\bullet}x := \{y \mid y \; F \; x\}$ is called the set of **preconditions.**

$x^{\bullet} := \{y \mid x \; F \; y\}$ is called the set of **postconditions.**

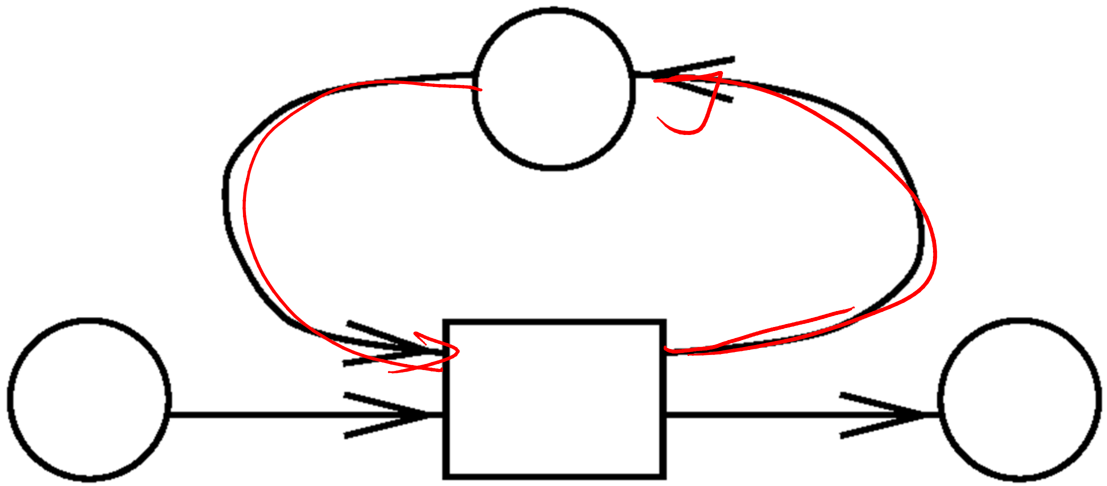
Example:



Basic structural properties: Loops and pure nets

REVIEW

Def.: Let $(c,e) \in C \times E$. (c,e) is called a **loop** iff $cFe \wedge eFc$.

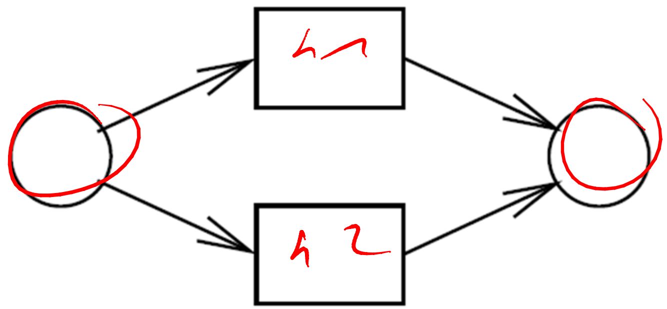


Def.: Net $N=(C,E,F)$ is called **pure**, if F does not contain any loops.

Simple nets

- **Def.:** A net is called **simple** if no two nodes $n1$ and $n2$ have the same pre-set and post-set.

- Example (not simple):



More complex example (1)

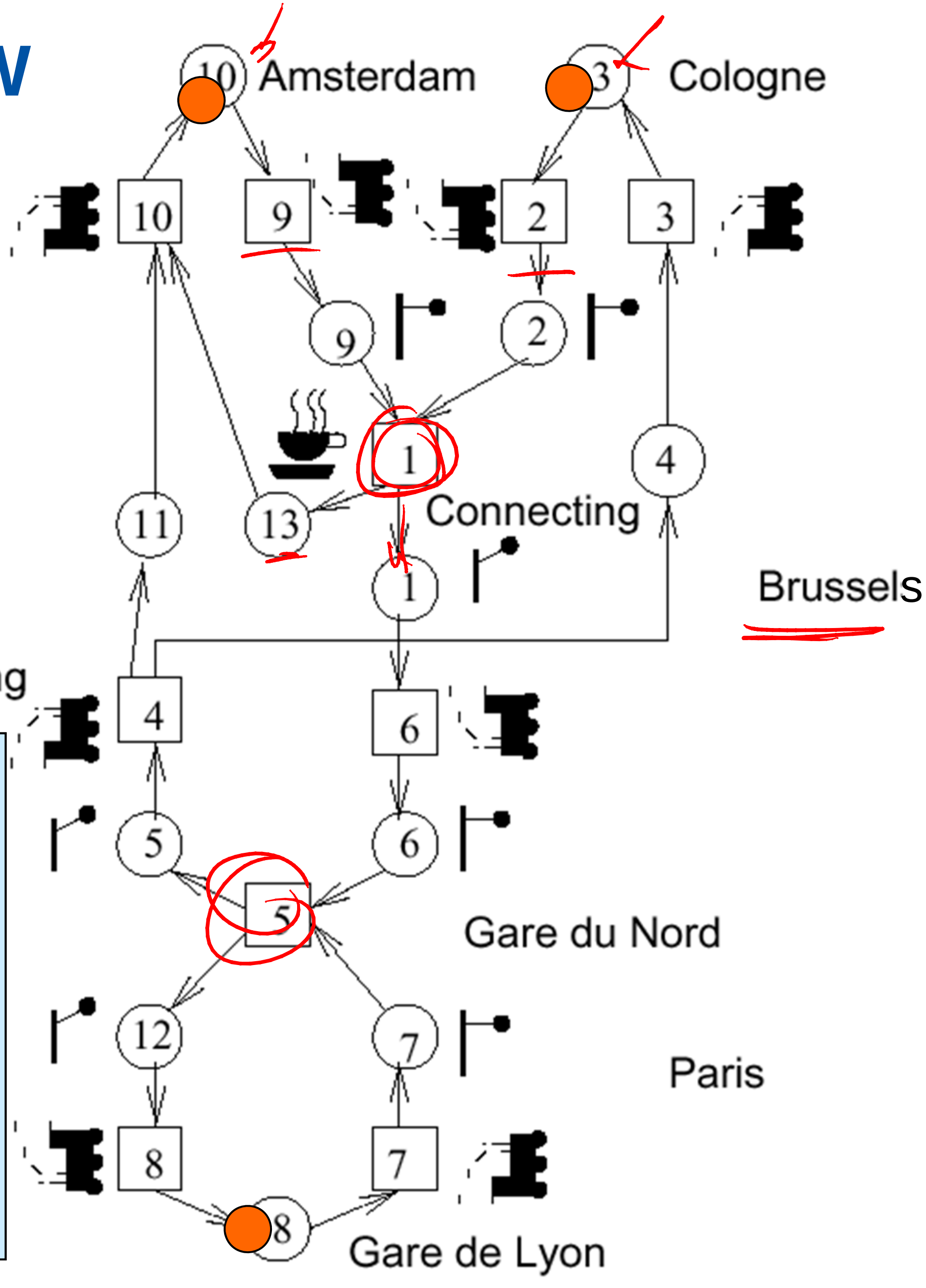
REVIEW

Thalys trains between Cologne, Amsterdam, Brussels and Paris.

[http://www.thalys.com/be/en]

Example Thalys trains: more complex **REVIEW**

- Thalys trains between Cologne, Amsterdam, Brussels and Paris.
- Synchronization at Brussels and Paris

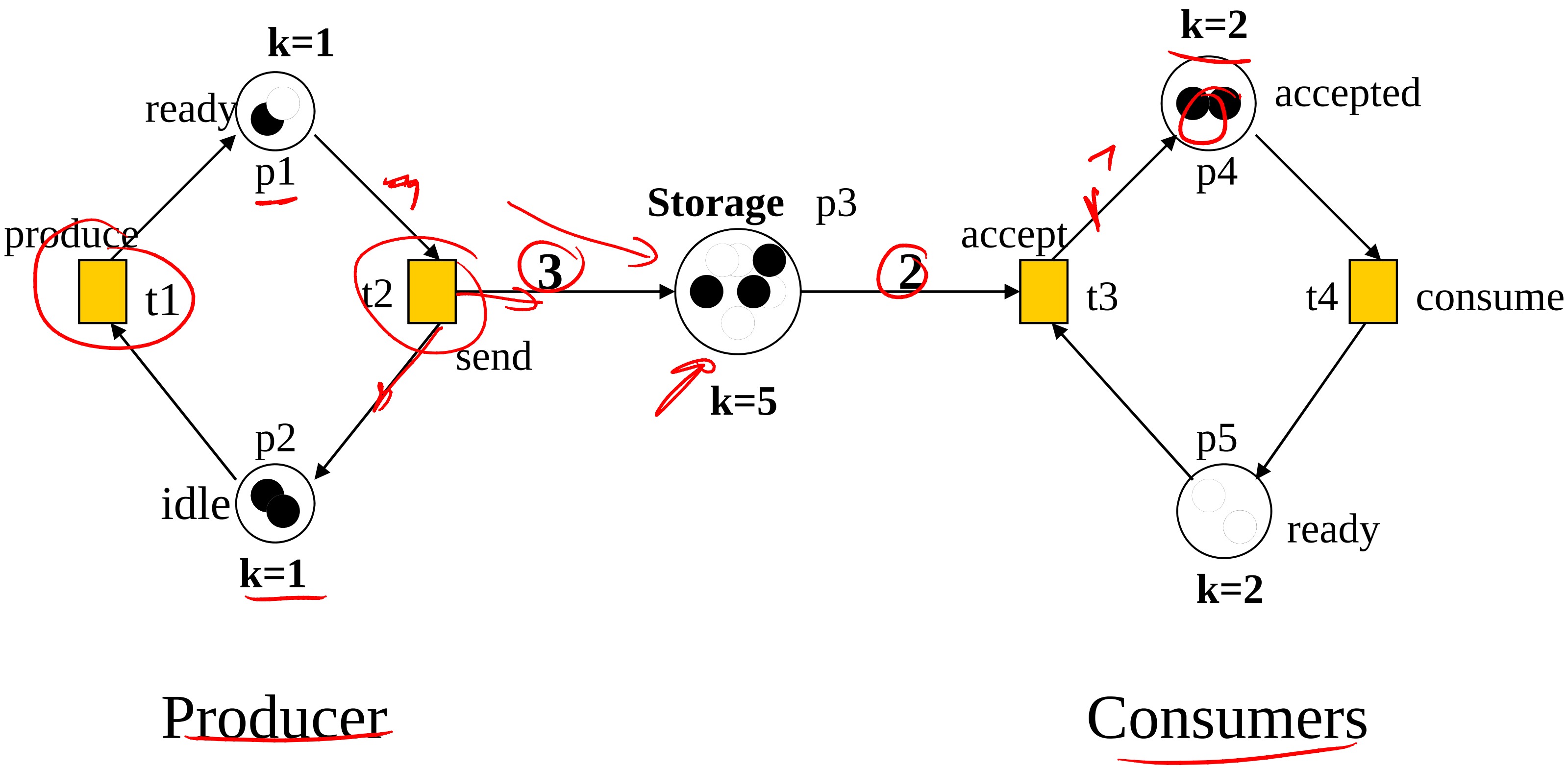


- Places 3 and10: trains waiting in A and C
- Transitions 9 and 2: trains driving from A and C to Brussels
- T1: connecting the two trains
- Break for driver P13
- T5 synchronization with ~~trians at~~ Gare du Nord

Realistic scenarios need more general definitions

REVIEW

- More than one token per condition, capacities of places
- weights of edges
- state space of Petri nets may become infinite!



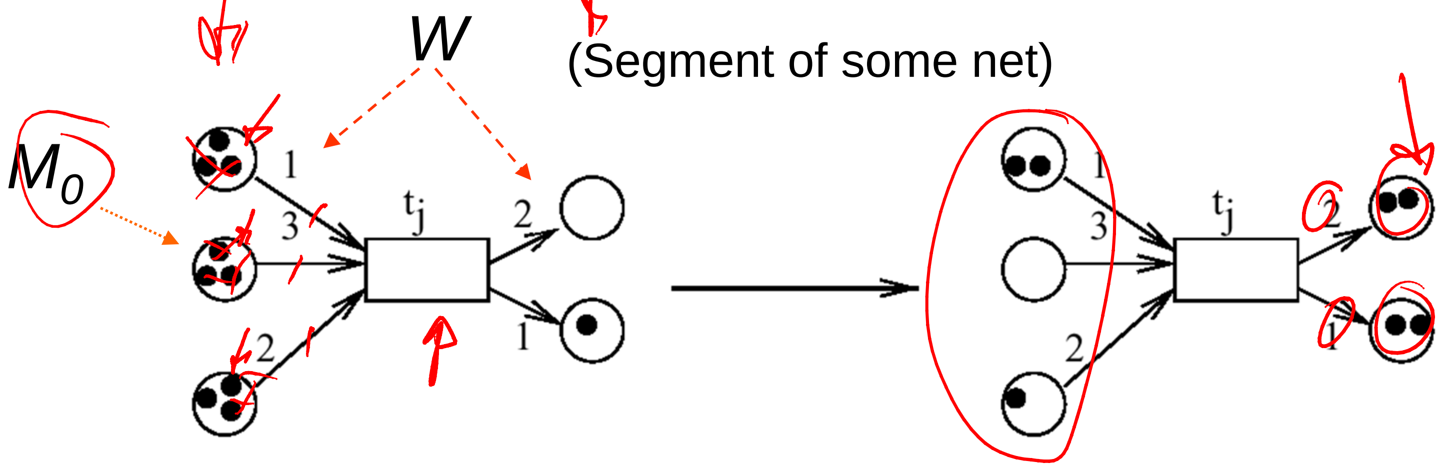
Place/transition nets

REVIEW

multiple tokens per place

Def.: (P, T, F, K, W, M_0) is called a **place/transition net (P/T net)** iff

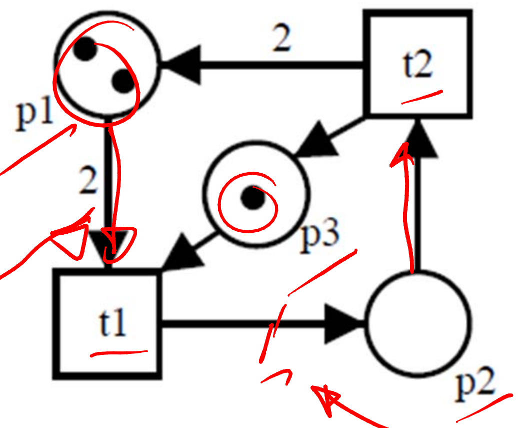
1. $N=(P,T,F)$ is a **net** with places $p \in P$ and transitions $t \in T$
2. $K: P \rightarrow (\mathbf{N_0} \cup \{\omega\}) \setminus\{0\}$ denotes the **capacity** of places
   (ω symbolizes infinite capacity)
3. $W: F \rightarrow (\mathbf{N_0} \setminus\{0\})$ denotes the **weight of graph edges**
4. $M_0: P \rightarrow \mathbf{N_0} \cup\{\omega\}$ represents the **initial marking** of places



defaults:
$K = \omega$
$W = 1$

Example

REVIEW

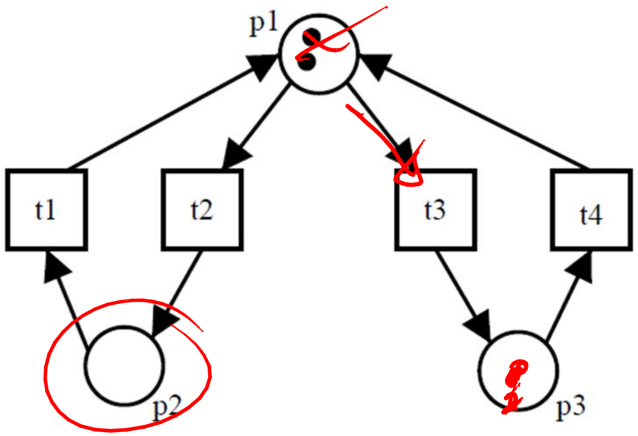


- P = {p1, p2, p3}
- T = {t1, t2}
- F = {(p1, t1), (p2, t2), (p3, t1), (t1, p2), (t2, p1), (t2, p3)}
- W = {(p1, t1) → 2, (p2, t2) → 1, (p3, t1) → 1, (t1, p2) → 1,
  (t2, p1) → 2, (t2, p3) → 1}
- m0 = (2, 0, 1)

  p1 p2 p3

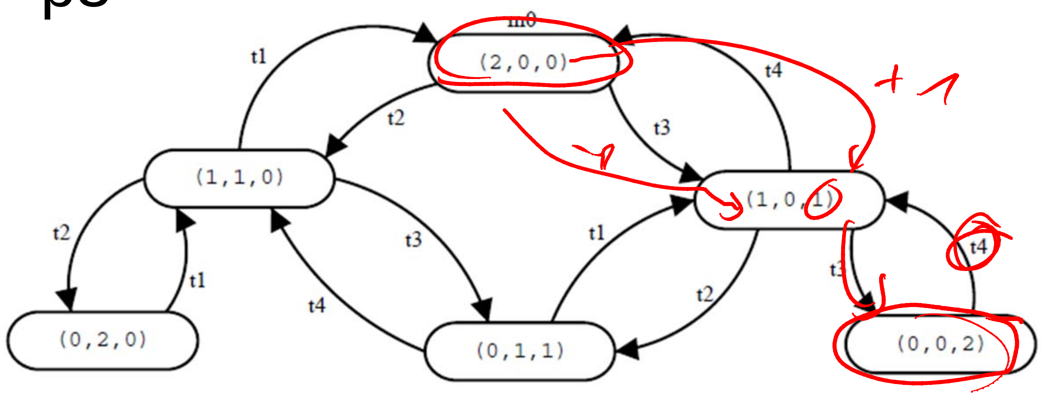
Reachability

REVIEW



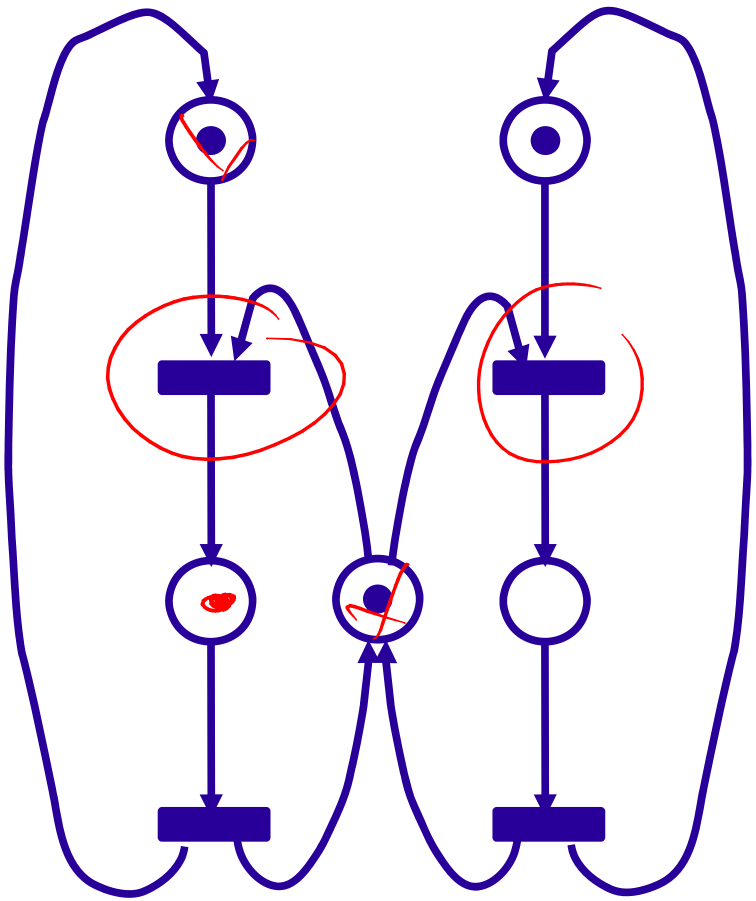
m0 = (2, 0, 0)

p1 p2 p3



reachability graph

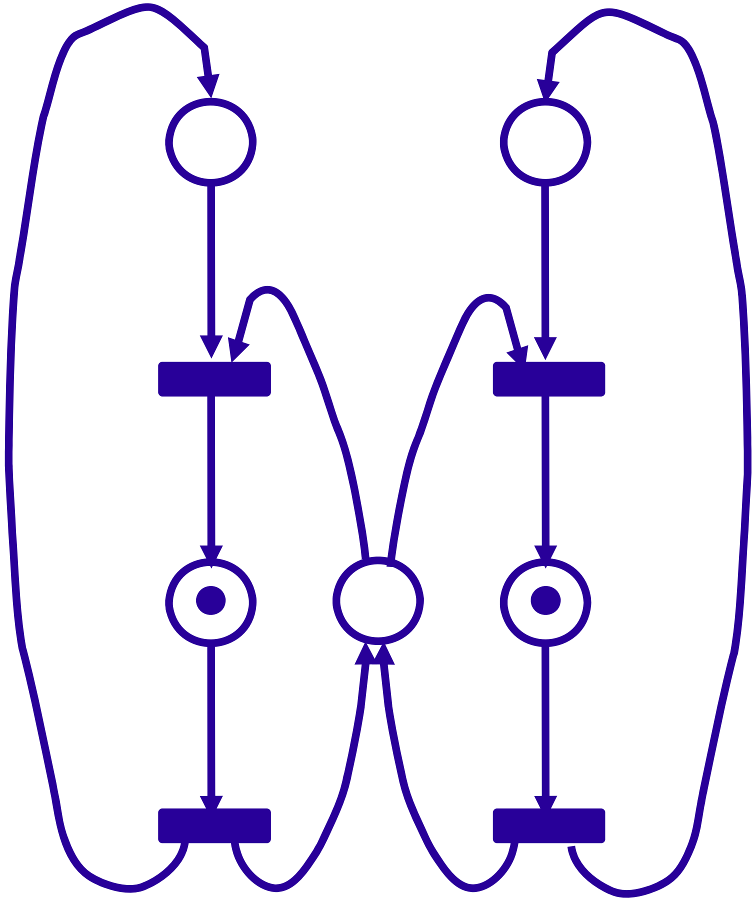
Reachability



Marking
M

Is there a sequence of transition firings such that $M \longrightarrow M'$?

NO



Marking
M'

REVIEW

From conditions to resources (1)

- c/e-systems model the flow of information, at a fundamental level (true/false)
- there are natural application areas for which the flow/transport of resources and the number of available resources is important (data flow, document-/workflow, production lines, communication networks, ..)
- place/transition-nets are a suitable generalisation of c/e-systems:
  - state elements represent places where resources (tokens) can be stored
  - transition elements represent local transitions or transport of resources

REVIEW

From conditions to resources (2)

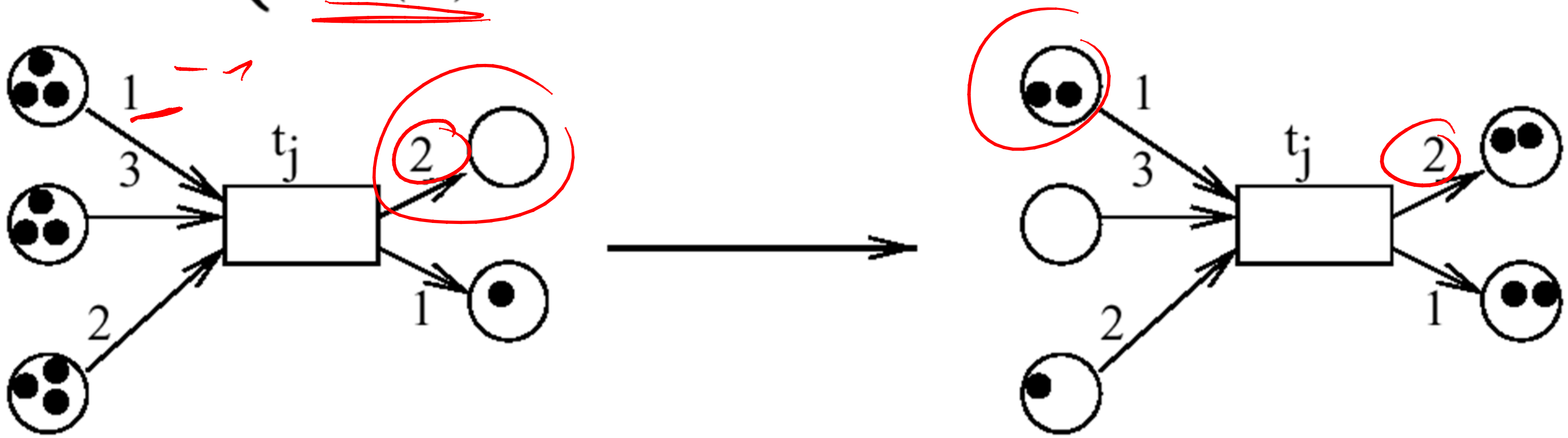
- a transition is enabled if and only if
  - sufficient resources are available on all its input places
  - sufficient capacities are available on all its output places

- a transition occurrence
  - consumes one token from each input place and
  - produces one token on each output place

Computing changes of markings

- „Firing“ transitions *t* generate new markings on each of the places *p* according to the following rules:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

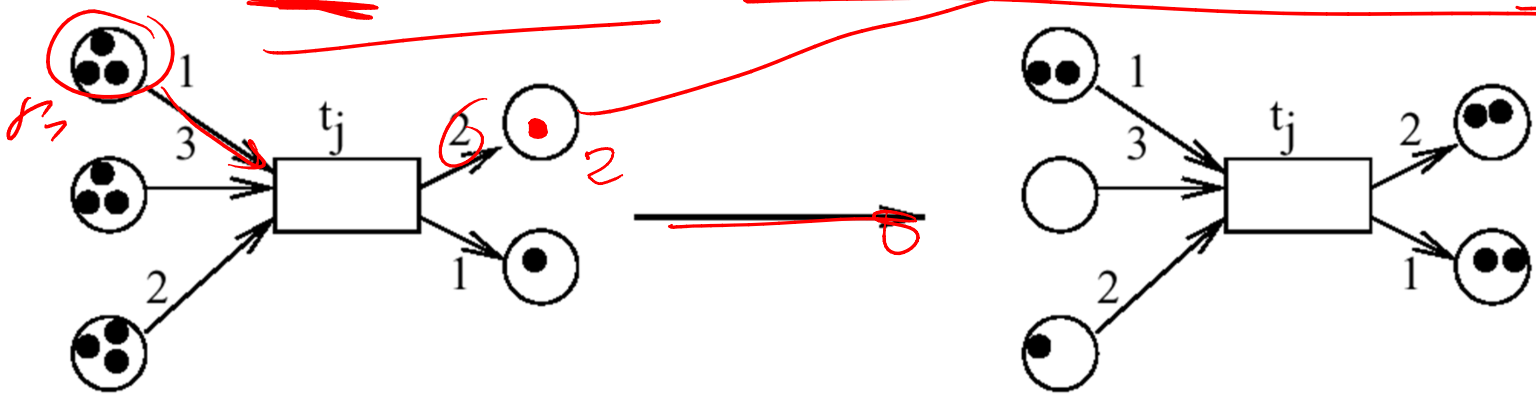


When a transition t *fires* from a marking M, w(p, t) tokens are deleted from the incoming places of t (i.e. from places p ∈ $^\bullet t$), and w(t, p) tokens are added to the outgoing places of t (i.e. to places p ∈ $t^\bullet$), and a new marking M' is produced

Activated transitions

- Transition *t* is „activated“ iff

$$(\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)) \wedge (\forall p \in t^{\bullet} : M(p) + W(t,p) \leq K(p))$$



Activated transitions can „take place“ or „fire“,
but don‘t have to.
The order in which activated transitions fire is not fixed
(it is non-deterministic).

Boundedness

- A place is called ***k-safe*** or ***k-bounded*** if it contains in the initial marking m_0 and in all other reachable from there markings at most **k** tokens.

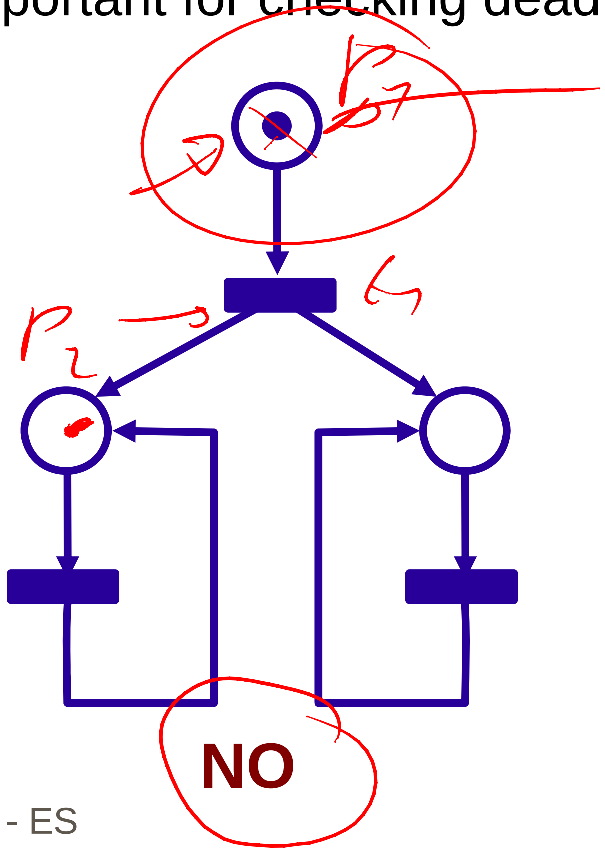
- A net is **bounded** if each place is bounded.

- **Boundedness: the number of tokens in any place cannot grow indefinitely**
- **Application: places represent buffers and registers (check there is no overflow)**

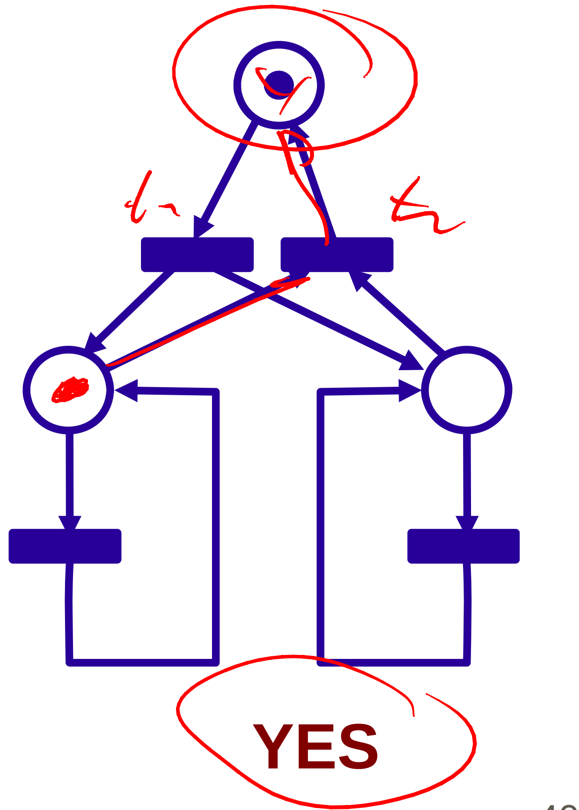
- A Petri net is inherently bounded if and only if all its reachability graphs (i.e. reachability graphs with all possible starting states) all have a finite number of states.

Liveness

- A transition T is live if in any marking there exists a firing sequence such that T becomes enabled
- An entire net is live if all its transitions are live
- Important for checking deadlock

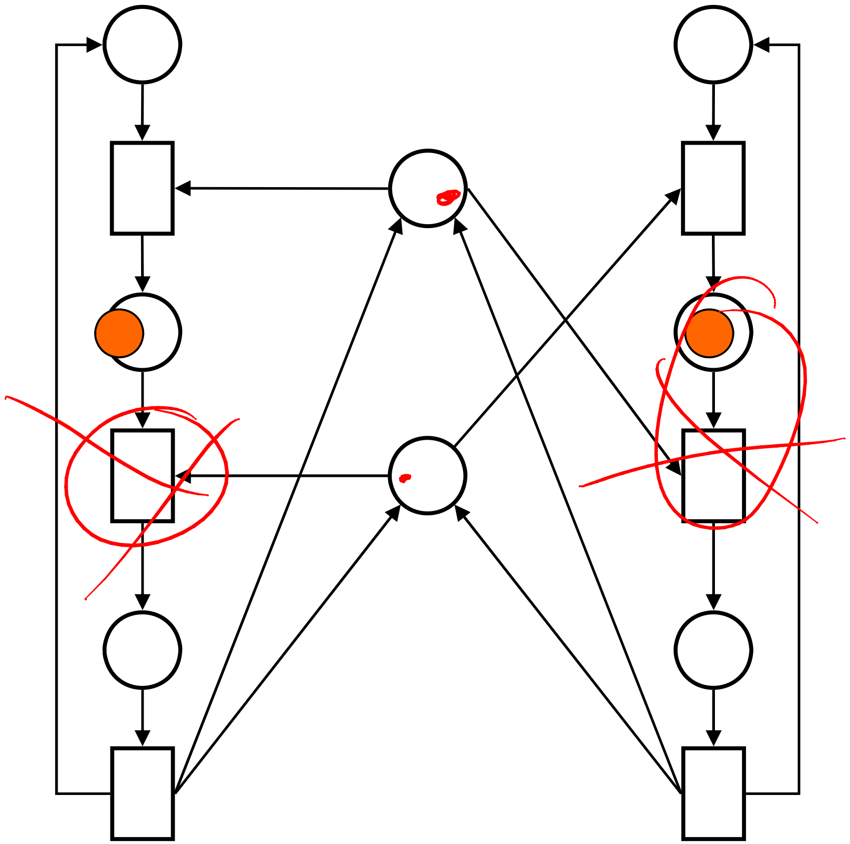


Live ?



Deadlock

- A **dead marking (deadlock)** is a marking where no transition can fire.
- A Petri net is **deadlock-free** if no dead marking is reachable.



Shorthand for changes of markings

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $M' = M + \underline{t}$

+: vector add

Matrix $\underline{N}$ describing all changes of markings

$$\underline{t}(p)=\begin{cases}-W(p,t)\text{ if } p\in{}^{\bullet}t\setminus t^{\bullet}\\ +W(t,p)\text{ if } p\in t^{\bullet}\setminus{}^{\bullet}t\\ -W(p,t)+W(t,p)\text{ if } p\in t^{\bullet}\cap{}^{\bullet}t\\ 0\end{cases}$$

Def.: Matrix $\underline{N}$ (incidence matrix) of net N is a mapping

$$\underline{N}: P\times T \rightarrow Z \text{ (integers)}$$

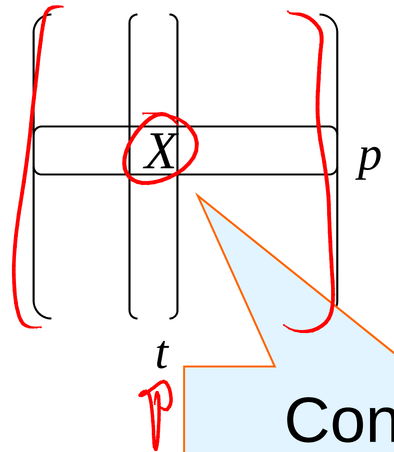
such that $\forall\, t\in T$: $\underline{N}(p,t)=\underline{t}(p)$

Component in column t and row p indicates the change of the marking of place p if transition t takes place.

Incidence matrix

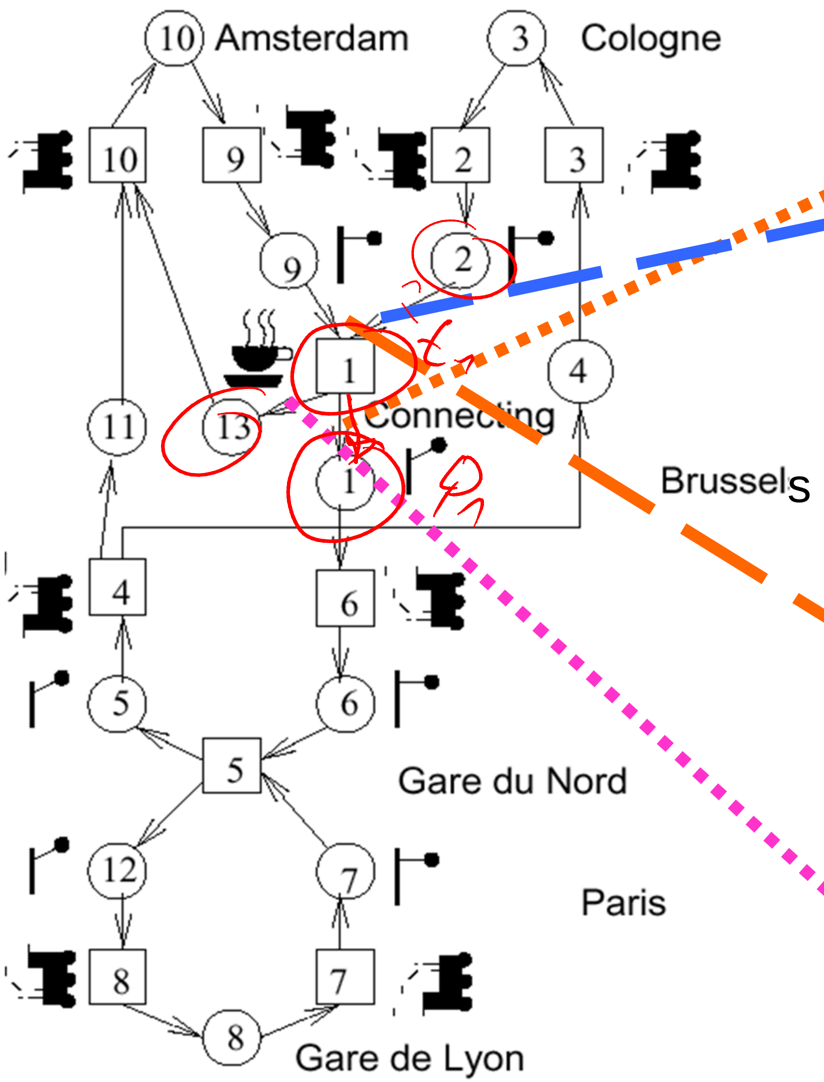
incidence matrix N of a pure (no elementary loops) place/transition-net:

$$N_{p,t} := \begin{cases} -W(t,p), & arc \text{ from } p \text{ to } t \\ +W(t,p), & arc \text{ from } t \text{ to } p \\ 0, & \text{otherwise} \end{cases}$$



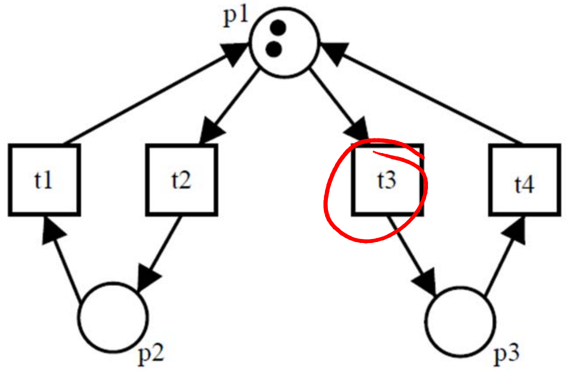
Contribution of *t* on *p*

Example: <u>*N*</u> =



	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
p_1	1					–1				
p_2	–1	1								
p_3		–1	1							
p_4			–1	1						
p_5				–1	1					
p_6					–1	1				
p_7					–1		1			
p_8							–1			
p_9	–1							1	1	
p_{10}									–1	1
p_{11}				1						–1
p_{12}					1			–1		
p_{13}	1									–1

State equation



$$N_{p,t} = \begin{array}{c|cccc} & t_1 & t_2 & t_3 & t_4 \\ \hline p_1 & 1 & -1 & -1 & 1 \\ p_2 & -1 & 1 & 0 & 0 \\ p_3 & 0 & 0 & 1 & -1 \end{array}$$

$$m_0 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

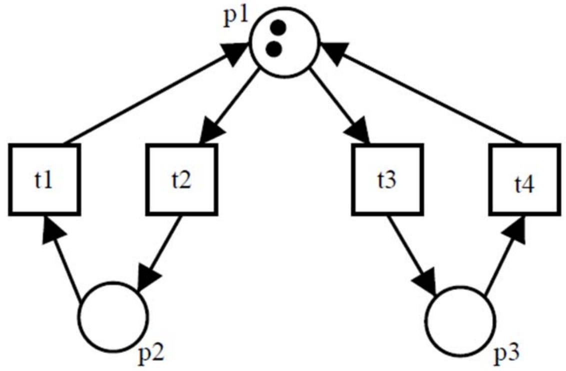
$$m' = m_0 + N \cdot u_i$$

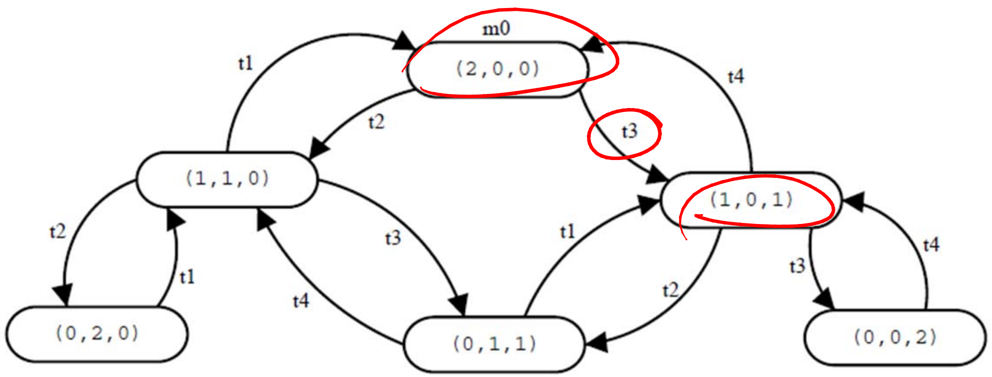
$$u_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ for } t_3$$

$$m' = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

State equation





reachability graph

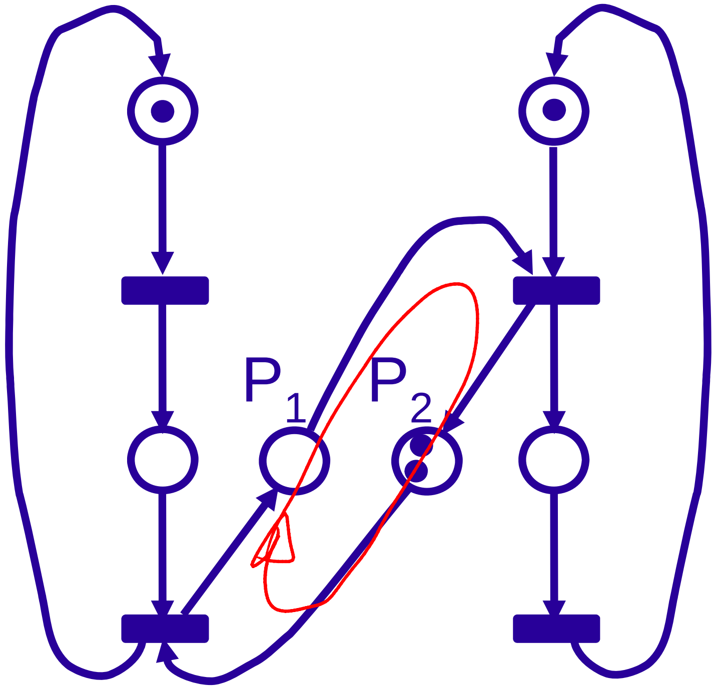
Computation of Invariants

We are interested in subsets *R* of places whose number of labels remain invariant under fireing of transitions:

- e.g. the number of trains commuting between Amsterdam and Paris (Cologne and Paris) remains constant

Important for correctness proofs

Computation of Invariants



$$P_1 + P_2 = 2$$

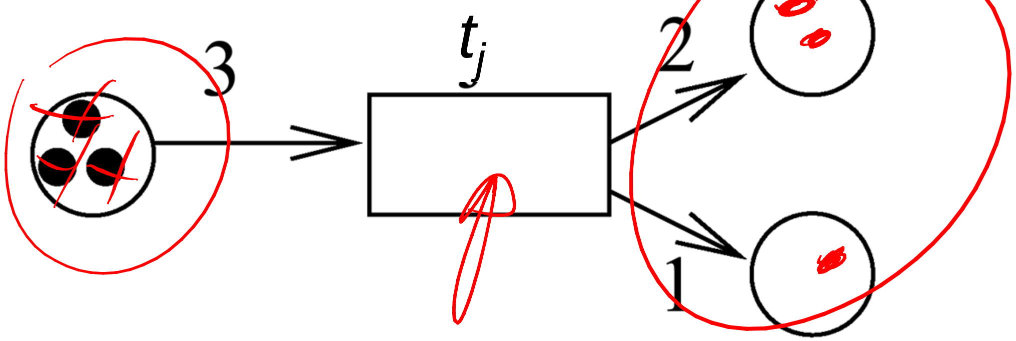
Place - invariants

Standardized technique for proving properties of system models

For any transition $t_j \in T$ we are looking for sets $R \subseteq P$ of places for which the accumulated marking is constant:

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Example:



Characteristic Vector

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Let:

$$\underline{c}_R(p) = \begin{cases} 1 \text{ if } & p \in R \\ 0 \text{ if } & p \notin R \end{cases}$$

$$\Rightarrow \quad \sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Scalar product

Condition for place invariants

$$\sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Accumulated marking constant for **all** transitions if

$$\begin{array}{lcl} \underline{t}_1 \cdot \underline{c}_R & = & 0 \\ \dots & \dots & \dots \\ \underline{t}_n \cdot \underline{c}_R & = & 0 \end{array}$$

Equivalent to $\underline{\boldsymbol{N}}^T \underline{\boldsymbol{c}}_R = \boldsymbol{0}$ where $\underline{N}^T$ is the transposed of $\underline{N}$

More detailed view of computations

$$\overbrace{\begin{pmatrix} t_1(p_1)\ldots t_1(p_n) \\ t_2(p_1)\ldots t_2(p_n) \\ \ldots \\ t_m(p_1)\ldots t_m(p_n) \end{pmatrix}}^{L} \underbrace{\begin{pmatrix} c_R(p_1) \\ c_R(p_2) \\ \ldots \\ c_R(p_n) \end{pmatrix}}_{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

System of linear equations.

Solution vectors must consist of zeros and ones.

Different techniques for solving equation system (Gauss elimination, tools e.g. Matlab, …)

Application to Thalys example

$\underline{N}^T \, \underline{c}_R = 0$, with $\underline{N}^T =$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}
t_1	1	-1							-1				1
t_2		1	-1										
t_3			1	-1									
t_4				1	-1						1		
t_5					1	-1	-1					1	
t_6	-1					1							
t_7							1	-1					
t_8								1				-1	
t_9									1	-1			
t_{10}										1	-1		-1

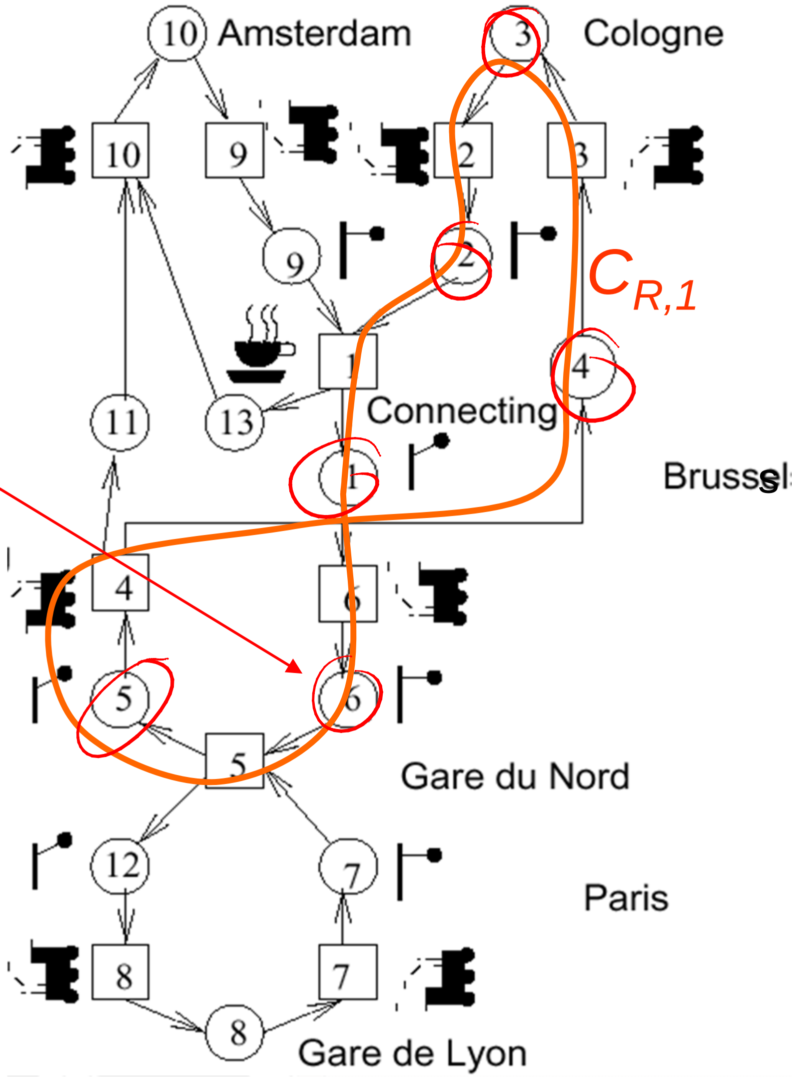
$$c_{R,1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Interpretation of the 1st invariant

$$c_{R,1} = \left(1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right)$$

Characteristic vector describes places for Cologne train.

We proved that: the number of trains along the path remains constant.



Application to Thalys example

$\underline{N}^T\ \underline{c}_R$ **= 0,** with $\underline{N}^T$=

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}
t_1	1	-1							-1				1
t_2		1	-1										
t_3			1	-1									
t_4				1	-1						1		
t_5					1	-1	-1					1	
t_6	-1					1							
t_7							1	-1					
t_8								1				-1	
t_9									1	-1			
t_{10}										1	-1		-1

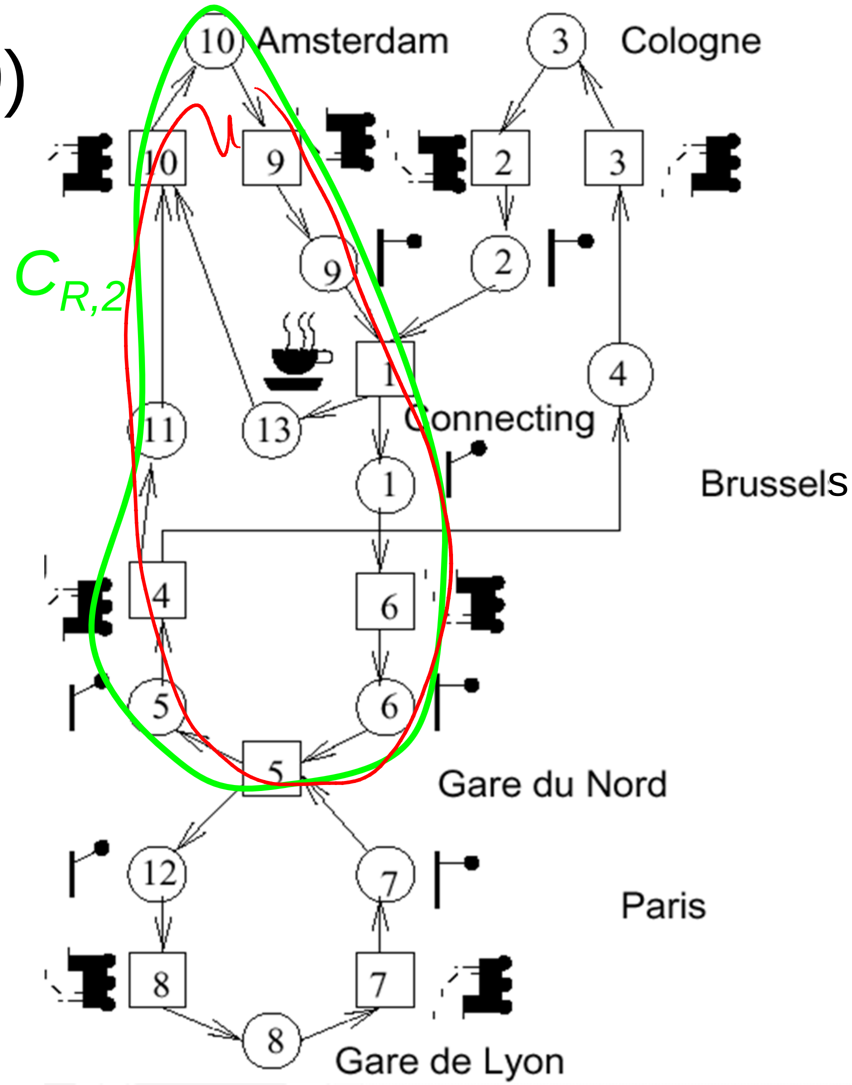
$$c_{R,2} = (1,0,0,0,1,1,0,0,1,1,1,0,0)$$

Interpretation of the 2nd invariant

$$c_{R,2} = (1,0,0,0,1,1,0,0,1,1,1,0,0)$$

We proved that:

None of the Amsterdam trains gets lost.



Application to Thalys example

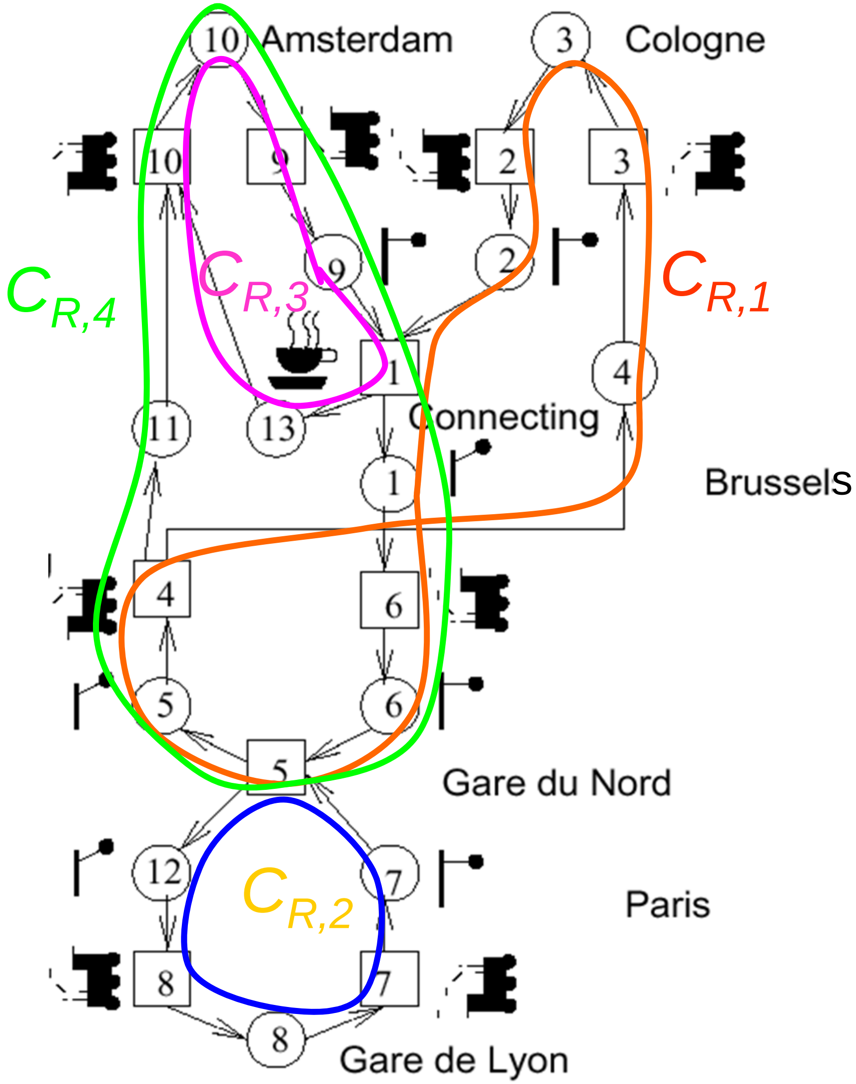
$\underline{\boldsymbol{N}}^T \, \underline{\boldsymbol{c}}_R = \mathbf{0}$, with $\underline{\boldsymbol{N}}^T =$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}
t_1	1	-1							-1				1
t_2		1	-1										
t_3			1	-1									
t_4				1	-1						1		
t_5					1	-1	-1					1	
t_6	-1					1							
t_7							1	-1					
t_8								1				-1	
t_9									1	-1			
t_{10}										1	-1		-1

$$c_{R,2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Solution vectors for Thalys example

$$c_{R,1} = (1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$$
$$c_{R,2} = (0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0)$$
$$c_{R,3} = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1)$$
$$c_{R,4} = (1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0)$$



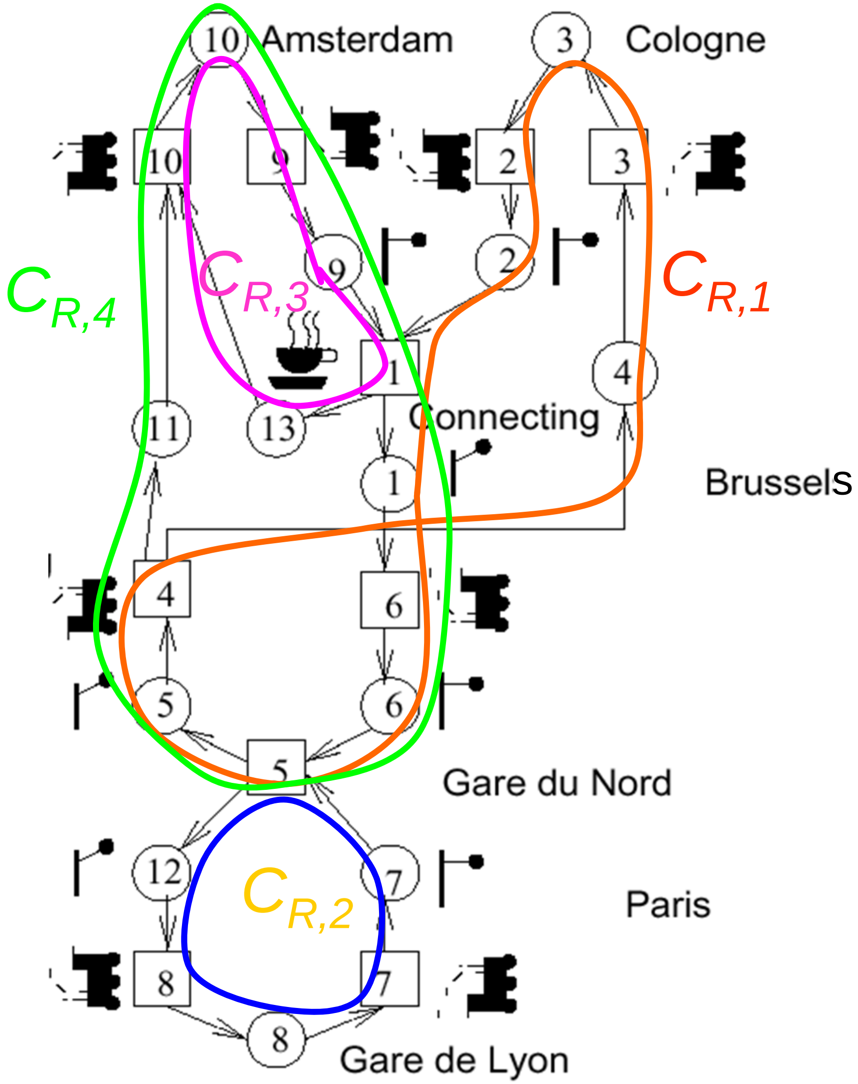
We proved that:

- the number of trains serving Amsterdam, Cologne and Paris remains constant.
- the number of train drivers remains constant.

Solution vectors for Thalys example

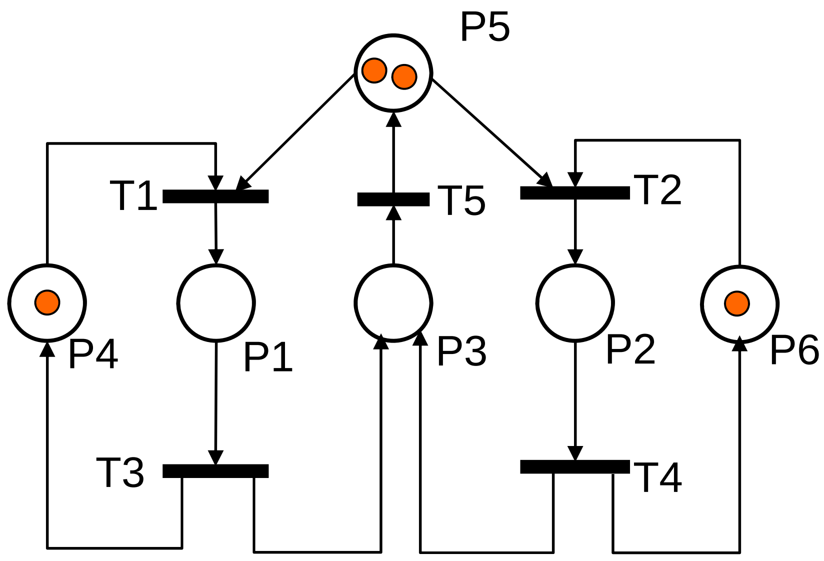
It follows:

- each place invariant must have at least one label at the beginning, otherwise “dead”
- at least three labels are necessary in the example



Place Invariants – Animation

http://www.informatik.uni-hamburg.de/TGI/PetriNets/introductions/aalst/trafficlight2_PI.swf



$\underline{\boldsymbol{N}}^T \underline{\boldsymbol{c}}_R = \mathbf{0}$, with $\underline{\boldsymbol{N}}^T$=

	P1	P2	P3	P4	P5	P6
T1	1	0	0	-1	-1	0
T2	0	1	0	0	-1	-1
T3	-1	0	1	1	0	0
T4	0	-1	1	0	0	1
T5	0	0	-1	0	1	0