Embedded Systems
Overview of embedded systems design

- Application knowledge
  - HW-components
  - Specification
  - Standard software (RTOS, ...)

- Hardware design
  - Implementation: hw/sw cosdesign
    - Task concurrency management
    - High-level transformations
    - Design space exploration
    - Hardware/software partitioning
    - Compilation, scheduling

- Realization
  - Software

- Validation; evaluation (performance, energy consumption, safety, ..)
Scheduling

- Support for multi-tasking/multi-threading – several tasks to run on shared resources
- **Task** ~ process – sequential program
- **Resources**: processor(s) + memory, disks, buses, communication channels, etc.
- Scheduler assigns **shared resources** to tasks for **durations of time**
- Most important resource(s) – **processor(s)**
- **Scheduling** – mostly concerned with processor(s)
  - **Online** – scheduling decisions taken when input becomes available
  - **Offline** – schedule computed with complete input known
- Other shared resources with exclusive access complicate scheduling task
Point of departure: Scheduling general IT systems

- In general IT systems, not much is known about the set of tasks a priori
  - The set of tasks to be scheduled is **dynamic**:
    - new tasks may be inserted into the running system,
    - executed tasks may disappear.
    - Tasks are activated with unknown activation patterns.
  - The power of schedulers thus is inherently limited by lack of knowledge – only **online scheduling** is possible
Scheduling processes in ES: The difference in process characterization

- Most ES are “closed shops”
  - Task set of the system is known
  - at least part of their activation patterns is known
    - periodic activation in, e.g., signal processing
    - maximum activation frequencies of asynchronous events determinable from environment dynamics, minimal inter-arrival times
  - Possible to determine bounds on their execution time (WCET)
    - if they are well-built
    - if we invest enough analysis effort
- Much better prospects for guaranteeing response times and for delivering high-quality schedules!
Scheduling – Time

- **Time** – aspect of the controlled plant/environment
- Embedded real-time systems have **deadlines** for their reactions, dictated by their environment
  - **hard deadline**: must be met, otherwise system is faulty, examples: airbag, ABS,
  - **soft deadline**: missing it decreases value of the result, examples: video transmission

- **Difference between**
  - **speed**: being fast on the average - and
  - **punctuality**: being always on time
    - Example: DB trains
Scheduling processes in ES: Differences in goals

- In classical OS, quality of scheduling is normally measured in terms of performance:
  - Throughput, reaction times, … in the average case
- In ES, the schedules do often have to meet stringent quality criteria under all possible execution scenarios:
  - A task of an RTOS is usually connected with a **deadline**. Standard operating systems do not deal with deadlines.
  - Scheduling of an RTOS has to be **predictable**.
  - Real-time systems have to be designed for **peak load**. Scheduling for meeting deadlines should work for all anticipated situations.
Constraints for real-time tasks

- Three types of constraints for real-time tasks:
  - Timing constraints
  - Precedence constraints
  - Mutual exclusion constraints on shared resources

- Typical timing constraints: **Deadlines** on tasks
  - **Hard**: Not meeting the deadline can cause catastrophic consequences on the system
  - **Soft**: Missing the deadline decreases performance of the system, but does not prevent correct behavior
Timing constraints and schedule properties

- Timing parameters of a real-time task $J_i$:
  - **Arrival time** $a_i$: time at which task becomes ready for execution
  - **Computation time** $C_i$: time necessary to the processor for executing the task without interruption
  - **Deadline** $d_i$: time before which a task should complete
  - **Start time** $s_i$: time at which a task starts its execution
  - **Finishing time** $f_i$: time at which a task finishes its execution
  - **Lateness** $L_i$: $L_i = f_i - d_i$, delay of task completion with respect to deadline
  - **Exceeding time** $E_i$: $E_i = \max(0, L_i)$
  - **Slack time** $X_i$: $X_i = d_i - a_i - C_i$, maximum time a task can be delayed on its activation to complete within its deadline
Timing parameters

- Additional timing related parameters of a real-time task $J_i$:
  - **Criticality**: parameter related to the consequences of missing the deadline
  - **Value $v_i$**: relative importance of the task with respect to other tasks in the system

- **Regularity of activation**:
  - **Periodic tasks**: Infinite sequence of identical activities (instances, jobs) that are regularly activated at a constant rate, here abbreviated by $\tau_i$
  - **A-periodic tasks**: Tasks which are not recurring or which do not have regular activations, here abbreviated by $J_i$
**Timing constraints of periodic tasks**

- **Phase** $\Phi_i$: activation time of first periodic instance
- **Period** $T_i$: time difference between two consecutive activations
- **Relative deadline** $D_i$: time after activation time of an instance at which it should be complete

![Diagram showing timing constraints of periodic tasks](image)
Scheduling - Basic definitions

Given a set of tasks \( \{J_1, J_2, \ldots, J_n\} \).

What do we require from a schedule?

- Every processor is assigned to at most one task at any time.
- Every task is assigned to at most one processor at any time.
- All the scheduling constraints are satisfied.

Def.: A (single-processor) schedule is a function \( \sigma : \mathbb{R}^+ \rightarrow \mathbb{N} \) such that

\[
8 \ t \in \mathbb{R}^+ \ 9 \ t_1 < t_2 \in \mathbb{R}^+ \cdot t \in [t_1, t_2) \text{ and } 8 \ t' \in [t_1, t_2) \ \sigma(t) = \sigma(t').
\]

In other words: \( \sigma \) is an integer step function and \( \sigma(t) = k \), with \( k > 0 \), means that task \( J_k \) is executed at time \( t \), while \( \sigma(t) = 0 \) means that the CPU is idle.

- A schedule is feasible, if all tasks can be completed according to a set of specified constraints.
- A set of tasks is schedulable if there exists at least one feasible schedule.
- **Schedulability test** executed before execution to derive guarantee.
Scheduling Algorithms

- Classes of scheduling algorithms:
  - Preemptive, non-preemptive
    - Task may be interrupted or always runs to completion
  - Off-line / on-line
    - Schedule works on actual and incomplete or on complete information
  - Optimal / heuristic – solutions must be optimal or sub-optimal are determined by using heuristics to reduce effort
  - One processor / multi-processor

- We start with single-processor scheduling.
Example

- **Non-preemptive schedule of three tasks** $J_1$, $J_2$, and $J_3$:
Example

- Preemptive schedule of three tasks $J_1$, $J_2$, and $J_3$: 

![Diagram showing preemptive schedule of three tasks](image)
Scheduling non-periodic tasks
A-periodic scheduling

- **Given:**
  - A set of a-periodic tasks \( \{J_1, \ldots, J_n\} \) with
    - arrival times \( a_i \), deadlines \( d_i \), computation times \( C_i \)
    - precedence constraints
    - resource constraints
  - Class of scheduling algorithm:
    - Preemptive, non-preemptive
    - Off-line / on-line
    - Optimal / heuristic
    - One processor / multi-processor
    - ...
  - Cost function:
    - Minimize maximum lateness (soft RT)
    - Minimize maximum number of late tasks (feasibility! – hard RT)

- **Find:**
  Optimal / good schedule according to given cost function
A-periodic scheduling

- Not all combinations of constraints, class of algorithm, cost functions can be solved efficiently.
- If there is some information on restrictions wrt. class of problem instances, then this information should be used!
- Begin with simpler classes of problem instances, then more complex cases.
Case 1: Aperiodic tasks with synchronous release

- A set of (a-periodic) tasks \{J_1, \ldots, J_n\} with
  - arrival times \(a_i = 0 \ 8 \ 1 \cdot i \cdot n\), i.e. “synchronous” arrival times
  - deadlines \(d_i\)
  - computation times \(C_i\)
  - no precedence constraints, no resource constraints, i.e. “independent tasks”

- non-preemptive

- single processor

- Optimal

- Find schedule which minimizes maximum lateness (variant: find feasible solution)
Preemption

**Lemma:**
If arrival times are synchronous, then preemption does not help, i.e. if there is a preemptive schedule with maximum lateness $L_{\text{max}}$, then there is also a non-preemptive schedule with maximum lateness at most $L_{\text{max}}$. 

![Diagram of task scheduling with notes:](image)

- Y3 same lateness
- All other may only have smaller lateness
- Interchange steps
- Termination: only finitely many steps
EDD – Earliest Due Date

EDD: execute the tasks in order of non-decreasing deadlines

- Example 1:

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cᵢ</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>dᵢ</td>
<td>3</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>
Example 2:

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$d_i$</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

$J_4$ misses its deadline.

Infeasible schedule.
Theorem (Jackson ’55):
Given a set of $n$ independent tasks with synchronous arrival times, any algorithm that executes the tasks in order of non-decreasing deadlines is optimal with respect to minimizing the maximum lateness.

Remark: Minimizing maximum lateness includes finding a feasible schedule, if it exists. The reverse is not necessarily true.
\[ L'_{\text{max}} = \max \{ f'_a - d_a, f'_b - d_b \} \]

\[ f'_a - d_a < f'_a - d_b \]

\[ f'_b - d_b \leq f'_b - d_b \]

\[ L'_{\text{max}} \leq L'_{\text{max}} \]
**EDD**

- **Complexity of EDD scheduling:**
  - Sorting $n$ tasks by increasing deadlines
  - $O(n \log n)$

- **Test of Schedulability:**
  If the conditions of the EDD algorithm are fulfilled, schedulability can be checked in the following way:

  - **Sort** task wrt. non-decreasing deadline.
    Let w.l.o.g. $J_1, \ldots, J_n$ be the sorted list.
  - Check whether in an EDD schedule $f_i \cdot d_i \ 8 \ i = 1, \ldots, n$.
  - Since $f_i = \sum_{k=1}^{i} C_k$, we have to check $8 \ i = 1, \ldots, n \ \sum_{k=1}^{i} C_k \cdot d_i$

  - Since EDD is optimal, non-schedulability by EDD implies non-schedulability in general.
Optimality Proofs for Scheduling Algorithms

Claim: Scheduling algorithm $A$ is optimal

- **Feasibility:** If exists a feasible schedule $S$ by some scheduling alg.
  Then: there exists a feasible schedule $S_A$ as obtained by $A$

- **Optimality:** If exists a schedule $S$ optimal w.r.t property $Q$ by some scheduling alg.
  Then: there exists a schedule $S_A$ as obtained by $A$ with property no worse than $Q$.

- **Proof technique:**

  - **Transform** schedule $S$ into a schedule $S_A$
    - Preserving feasibility
    - Not impairing property $Q$

  - **Show this for each transformation step:**
    - Select a task/slice of a task in $S$ violating the criterion of $A$
    - Move it to a position satisfying this criterion
Case 2: aperiodic tasks with asynchronous release

- A set of (a-periodic) tasks \{J_1, \ldots, J_n\} with
  - arbitrary arrival times \(a_i\)
  - deadlines \(d_i\)
  - computation times \(C_i\)
  - no precedence constraints, no resource constraints, i.e.
    “independent tasks”

- preemptive

- Single processor

- Optimal

- Find schedule which minimizes maximum lateness
  (variant: find feasible solution)
EDF – Earliest Deadline First

- At every instant execute the task with the earliest absolute deadline among all the ready tasks.

- Remark:
  1. If a new task arrives with an earlier deadline than the running task, the running task is immediately preempted.
  2. Here we assume that the time needed for context switches is negligible – we’ll later see that this is unrealistic.
EDF - Example

<table>
<thead>
<tr>
<th></th>
<th>J_1</th>
<th>J_2</th>
<th>J_3</th>
<th>J_4</th>
<th>J_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_i</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>C_i</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>d_i</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

Schedule is feasible
EDF

- **Theorem (Horn ’74):**
  Given a set of $n$ independent tasks with arbitrary arrival times, any algorithm that at every instant executes the task with the earliest absolute deadline among all the ready tasks is optimal with respect to minimizing the maximum lateness.
Non-preemptive version

- Changed problem:
  - A set of (a-periodic) tasks \( \{J_1, \ldots, J_n\} \) with
    - *arbitrary* arrival times \( a_i \)
    - deadlines \( d_i \),
    - computation times \( C_i \)
    - no precedence constraints, no resource constraints, i.e. “independent tasks”

- **Non-preemptive** instead of preemptive scheduling!
- Single processor
- Optimal
- Find schedule which minimizes maximum lateness (variant: find feasible solution)
Example

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>aᵢ</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Cᵢ</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>dᵢ</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

- Non-preemptive EDF schedule:

- Optimal schedule:
Example

- Observation:
  - In the optimal schedule the processor remains idle in interval [0,1) although task $J_1$ is ready to execute.
  - If arrival times are not known a-priori, then no on-line algorithm is able to decide whether to stay idle at time 0 or to execute $J_1$.

- **Theorem** (Jeffay et al. ’91): EDF is an optimal non-idle scheduling algorithm also in a non-preemptive task model.
Non-preemptive scheduling: better schedules through the introduction of idle times

- Assumptions:
  - Arrival times known a priori.
  - Non-preemptive scheduling
  - “Idle schedules” are allowed.

- Goal:
  - Find feasible schedule

- Problem is NP-hard.

- Possible approaches:
  - Heuristics
  - Branch-and-bound
Bratley’s algorithm

- Bratley’s algorithm
  - Finds feasible schedule by branch-and-bound, if there exists one
  - Schedule derived from appropriate permutation of tasks $J_1, \ldots, J_n$
  - Starts with empty task list
  - Branches: Selection of next task (one not scheduled so far)
  - Bound:
    - Feasible schedule found at current path -> search path successful
    - There is some task not yet scheduled whose addition causes a missed deadline -> search path is blind alley
Bratley’s algorithm

- Example:

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
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<th>$J_3$</th>
<th>$J_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i$</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$C_i$</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<tr>
<td>$d_i$</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
Bratley’s algorithm

- Due to exponential worst-case complexity only applicable as off-line algorithm.

- Resulting schedule stored in task activation list.
- At runtime: dispatcher simply extracts next task from activation list.
Case 3: Scheduling with precedence constraints

- Non-preemptive scheduling with non-synchronous arrival times, deadlines and precedence constraints is NP-hard.

- Here:
  - Restrictions:
    - Consider synchronous arrival times (all tasks arrive at 0)
    - Allow preemption.
  - 2 different algorithms:
    - Latest deadline “first” (LDF)
    - Modified EDF

- Precedences define a partial order
- Scheduling determines a compatible total order
- Method: Topological sorting
Example

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
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<th>J₆</th>
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<tbody>
<tr>
<td>aᵢ</td>
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<td>5</td>
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</tr>
</tbody>
</table>

Diagram:

- J₁
- J₂
- J₃
- J₄
- J₅
- J₆
Example

One of the following algorithms is optimal. Which one?

Algorithm 1:
1. Among all sources in the precedence graph select the task T with earliest deadline. Schedule T first.
2. Remove T from G.
3. Repeat.

Algorithm 2:
1. Among all sinks in the precedence graph select the task T with latest deadline. Schedule T last.
2. Remove T from G.
3. Repeat.
Example (continued)

- Algorithm 1:

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
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<tbody>
<tr>
<td>aᵢ</td>
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<tr>
<td>dᵢ</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

```
1 0 0 0 0 0
1 1 1 1 1 1
2 5 4 3 5 6
```
Example (continued)

- Algorithm 2:

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
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</tbody>
</table>

\[\begin{array}{llllll}
2 & 5 & 4 & 3 & 5 & 6 \\
\end{array}\]

![Algorithm 2 diagram](image-url)
Example (continued)

- Algorithm 1 is **not** optimal.
- Algorithm 1 is the generalization of EDF to the case with precedence conditions.

- Is Algorithm 2 optimal?
- Algorithm 2 is called Latest Deadline First (LDF).

- **Theorem (Lawler 73):**
  LDF is optimal wrt. maximum lateness.
Proof of optimality
**LDF**

- LDF is optimal.
- LDF may be applied only as off-line algorithm.

**Complexity of LDF:**
- $O(|E|)$ for repeatedly computing the current set $\Gamma$ of tasks with no successors in the precedence graph $G = (V, E)$.
- $O(\log n)$ for inserting tasks into the ordered set $\Gamma$ (ordering wrt. $d_i$).
- Overall cost: $O(n \times \max(|E|, \log n))$