Embedded Systems
**Periodic scheduling**

**Given:**
- A set of periodic tasks $\Gamma = \{\tau_1, \ldots, \tau_n\}$ with
  - phases $\Phi_i$ (arrival times of first instances of tasks),
  - periods $T_i$ (time difference between two consecutive activations)
  - relative deadlines $D_i$ (deadline relative to arrival times of instances)
  - computation times $C_i$
  
$\Rightarrow j$th instance $\tau_{i,j}$ of task $\tau_i$ with
  - arrival time $a_{i,j} = \Phi_i + (j-1)T_i$,
  - deadline $d_{i,j} = \Phi_i + (j-1)T_i + D_i$,
  - start time $s_{i,j}$ and
  - finishing time $f_{i,j}$

**Find** a feasible schedule
Assumptions

A.1. Instances of periodic task $\tau_i$ are regularly activated with constant period $T_i$.
A.2. All instances have same worst case execution time $C_i$.
A.3. All instances have same relative deadline $D_i$, here in most cases equal to $T_i$ (i.e., $d_{i,j} = \Phi_i + j \cdot T_i$)
A.4. All tasks in $\Gamma$ are independent. No precedence relation, no resource constraints.
A.5. Overhead for context switches is neglected, i.e. assumed to be 0 in the theory.

- Basic results based on these assumptions form the core of scheduling theory.
- For practical applications, assumptions A.3. and A.4. can be relaxed, but results have to be extended.
Definition:
Given a set $\Gamma$ of $n$ periodic tasks, the processor utilization $U$ is given by

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}.$$ 

- Define $U_{\text{bnd}}(A) = \inf \{U(\Gamma) \mid \Gamma \text{ is not schedulable by algorithm A} \}$.

- If $U_{\text{bnd}}(A) > 0$ then a simple, sufficient criterion for schedulability by A can be based on processor utilization:
  - If $U(\Gamma) < U_{\text{bnd}}(A)$ then $\Gamma$ is schedulable by A.
  - However, if $U_{\text{bnd}}(A) < U(\Gamma) \leq 1$, then $\Gamma$ may or may not be schedulable by A.
Earliest Deadline First (EDF)

- **Theorem:** A set of periodic tasks $\tau_1, \ldots, \tau_n$ with $D_i = T_i$ is schedulable with EDF iff $U \cdot 1$.

- EDF is applicable to both periodic and a-periodic tasks.

- If there are only periodic tasks, priority-based schemes like “rate monotonic scheduling (RM)” (see later) are often preferred, since
  - They are simpler due to fixed priorities ⇒ use in “standard OS” possible
  - sorting wrt. to deadlines at run time is not needed
Rate monotonic scheduling (RM) (Liu, Layland ’73):

- Assign fixed priorities to tasks $\tau_i$:
  - $\text{priority}(\tau_i) = 1/T_i$
  - I.e., priority reflects release rate
- Always execute ready task with highest priority
- Preemptive: currently executing task is preempted by newly arrived task with shorter period.

- Theorem (Liu, Layland, 1973):
  RM is optimal among all fixed-priority scheduling algorithms.
Non-RM Schedule

Schedule feasible iff $C_1 + C_2 \leq T_1$
Let \( F = \left\lfloor \frac{T_2}{T_1} \right\rfloor \) be the number of periods of \( \tau_1 \) entirely contained in \( T_2 \).

**Case 1:**

- The computation time \( C_1 \) is short enough, so that all requests of \( \tau_1 \) within period of \( \tau_2 \) are completed before second request of \( \tau_2 \).
- I.e. \( C_1 \leq T_2 - FT_1 \)
- Schedule feasible if \((F+1)C_1 + C_2 \leq T_2\)
Case 2:

- The second request of $\tau_2$ arrives when $\tau_1$ is running.
- I.e. $C_1 \geq T_2 - F T_1$

Schedule feasible if $FC_1 + C_2 \leq FT_1$
Computation of $U_{bnd}(RM)$

- First step: Consider only task sets with 2 tasks
  - Computation of $U_{bnd}(RM, 2) = \inf \{U(\Gamma) \mid \Gamma \text{ is not schedulable by RM, } |\Gamma| = 2\}$. 
  - Idea:
    - Construct set of tasks with following properties:
      1. Set of tasks is schedulable by RM.
      2. Any increase of computation times makes the set of tasks non-schedulable.
      3. Processor utilization is minimal under properties 1. and 2.
    - “Worst case task set”
Computation of $U_{\text{bnd}}(RM, 2)$

- Worst case situation constructed for 2 processes:

![Diagram showing worst case situation for 2 processes with idle times marked]
Computation of $U_{bnd}(\text{RM}, 2)$

- Consider a set of 2 periodic tasks $\tau_1$ and $\tau_2$ with $T_1 \leq T_2$ $\Rightarrow$ $\text{priority}(\tau_1) > \text{priority}(\tau_2)$.
- We consider the critical instant when $\tau_1$ and $\tau_2$ arrive at the same time.
- We construct a worst case scenario where the processor utilization factor is minimal, but any increase of computation times destroys schedulability.

This is done by manipulating
- computation times $C_1$ and $C_2$ and
- $T_1$ and $T_2$ (more precisely $T_2 / T_1$)
Case 1: $C_1 \leq T_2 - F T_1$
Case 2: $C_1 \geq T_2 - F T_1$
Manipulating $T_2/T_1$
Computation of $U_{bnd}(RM)$

- Result for two processes:
  Any set of two periodic tasks with a processor utilization factor \( \leq U_{bnd} = 2(2^{1/2} - 1) \) can be scheduled by RM.

- Similarly, for the general case of \( n \) processes the following can be shown:
  Any set of \( n \) periodic tasks with a processor utilization factor \( \leq U_{bnd} = n(2^{1/n} - 1) \) can be scheduled by RM.
Computation of $U_{bnd}(RM)$

- Any set of $n$ periodic tasks with a processor utilization factor $\leq U_{bnd} = n(2^{1/n} - 1)$ can be scheduled by RM.

- Observation: $U_{bnd}$ is decreasing with $n$. 

![Graph showing decreasing values of $U_{bnd}$ with increasing $n$.]
Schedulability check

- To provide a lower bound for arbitrary \( n \) we have to compute
  \[ U_{lub} = n(2^{1/n} - 1) = \ln 2 \approx 0.69 \]

- Hence, a set of tasks can be scheduled by RM if
  \[ U < U_{bnd}(RM) = \ln 2 \approx 0.69 \]

- But what can we tell about **schedulability** when processor utilization factor is **larger than** \( n(2^{1/n} - 1) \)?

- Answer:
  We can compute a more precise result, if we make use of the knowledge of periods \( T_i \) and computation times \( C_i \).
Schedulability check

- Remember:
  The response time $R_{i,j}$ of an instance $j$ of task $i$ is the time (measured from the arrival time) at which the instance is finished: $R_{i,j} = f_{i,j} - a_{i,j}$.

- Compute an upper bound $R_i$ on the response time:
  - Suppose that $\tau_1, \ldots, \tau_n$ are ordered with increasing periods (i.e. decreasing priorities).
  - Consider an arbitrary periodic task $\tau_i$.
  - At a critical instant $t$, when an instance of $\tau_i$ arrives together with all higher priority tasks, we have:
    \[ R_i = C_i + \sum_{k=1}^{i-1} (# \text{ activations of } \tau_k \text{ during } [t, t + R_i]) \cdot C_k = C_i + \sum_{k=1}^{i-1} \left\lceil R_i/T_k \right\rceil \cdot C_k \]
Schedulability check

⇒ We just need to solve n fixed point equations to compute $R_1, ..., R_n$.
  - $R_i = C_i + \sum_{k=1}^{i-1} \left\lfloor \frac{R_i}{T_k} \right\rfloor \cdot C_k$
  - $R_i$ is the largest fixed point of the function $f(R_i) = C_i + \sum_{k=1}^{i-1} \left\lfloor \frac{R_i}{T_k} \right\rfloor \cdot C_k$.
  - Sufficient condition for schedulability: $R_i \leq D_i \ \forall \ i$

- Problem: Solution of fixed point equation?
Schedulability check

- Solution of fixed point equation?

- Compute the following sequence:
  - $R_i^{(0)} = C_i$.
  - $R_i^{(j+1)} = C_i + \sum_{k=1}^{i-1} \left\lfloor \frac{R_i^{(j)}}{T_k} \right\rfloor \cdot C_k$.

- It is easy to see that this sequence is monotonically increasing, i.e., $f(x) = C_i + \sum_{k=1}^{i-1} \left\lfloor \frac{x}{T_k} \right\rfloor \cdot C_k$ is monotonically increasing.

- $\Rightarrow$ If a least fixed point of $f(x)$ exists, then the sequence converges to this fixed point.
Schedulability check

**Lemma**: The sequence is unboundedly increasing or converges *after a finite number of steps* to the least fixed point.
Schedulability check

⇒ Algorithm:

∀ i: R_i^{(0)} = C_i
repeat
    ∀ i: R_i^{(j+1)} = C_i + \sum_{k=1}^{i-1} \left[ \frac{R_i^{(j)}}{T_k} \right] \cdot C_k
until (∃ i with R_i^{(j+1)} > D_i) or (∀ i R_i^{(j+1)} = R_i^{(j)});
if (∀ i R_i^{(j+1)} = R_i^{(j)}) then
    report(“RM schedulable”);
Summary

- Problem of scheduling independent and preemptable periodic tasks

- Rate monotonic scheduling:
  - Optimal solution among all fixed-priority schedulers
  - Schedulability of $n$ tasks guaranteed, if processor utilization $U \leq n(2^{1/n} - 1)$.

- Earliest deadline first:
  - Optimal solution among all dynamic-priority schedulers
  - Schedulability guaranteed if processor utilization $U \cdot 1$. 
Rate Monotonic Scheduling in Presence of Task Dependencies
Assumptions so far

A.1. Instances of periodic task $\tau_i$ are regularly activated with constant period $T_i$.
A.2. All instances have same worst case execution time $C_i$.
A.3. All instances have same relative deadline $D_i$, here in most cases equal to $T_i$ (i.e., $d_{i,j} = \Phi_i + j \cdot T_i$)
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Wait state caused by resource constraints

- Each mutually exclusive resource $R_i$ is protected by a semaphore $S_i$.
- Each critical section operating on $R_i$ must begin with a \textit{wait}(S$_i$) primitive and end with a \textit{signal}(S$_i$) primitive.
- \textit{wait} primitive on locked semaphore → wait state until another task executes signal primitive
The priority inversion problem

- **Priority inversion** can occur due to resource conflicts (exclusive use of shared resources) in fixed priority schedulers like RM:

![Diagram showing priority inversion]

- Here: Blocking time equal to length of critical section.
The priority inversion problem

- Blocking time equal to length of critical section + computation time of $J_2$.
- **Unbounded time of priority inversion**, if $J_3$ is interrupted by tasks with priority between $J_1$ and $J_3$ during its critical region.
Priority inversion in real life: The MARS Pathfinder problem (1)

“But a few days into the mission, not long after Pathfinder started gathering meteorological data, the spacecraft began experiencing total system resets, each resulting in losses of data. The press reported these failures in terms such as "software glitches" and "the computer was trying to do too many things at once"." …
Priority inversion in real life: The MARS Pathfinder problem (2)

- System overview:
  - *Information Bus (IB)*:
    - Buffer for exchanging data between different tasks
    - Shared resource of two tasks M and B

- Three tasks:
  - *Meteorological data gathering task (M)*:
    - collects meteorological data
    - reserves IB, writes data to IB, releases IB
    - infrequent task, low priority
  - *Bus management (B)*:
    - data transport from IB to destination
    - reserves IB, data transport, releases IB
    - frequent task, high priority
Priority inversion in real life:
The MARS Pathfinder problem (3)

- Three tasks:
  - ... 
  - “Communication task” (C):
    - medium priority, does not use IB

- Scheduling with fixed priorities.

- **Watch dog timer (W):**
  - Execution of B as indicator of system hang-up
  - If B is not activated for certain amount of time: Reset the system
Priority inversion in real life:
The MARS Pathfinder problem (5)

Reset by watchdog timer

J₁ blocked

J₁
J₂
J₃

B
C
M

normal execution  critical region

priority(J₁) > priority(J₂) > priority(J₃)
Coping with priority inversion: The priority inheritance protocol

Idea of priority inheritance protocol:

- If a task $J_h$ blocks, since another task $J_l$ with lower priority owns the requested resource, then $J_l$ inherits the priority of $J_h$.
- When $J_l$ releases the resource, the priority inheritance from $J_h$ is undone.
- Rule: Tasks always inherit the highest priority of tasks blocked by it.
Direct vs. push-through blocking

- **Direct blocking**: High-priority job tries to acquire resource already held by lower-priority job
- **Push-through blocking**: Medium-priority job is blocked by lower-priority job that has inherited a higher priority.
Transitive priority inheritance
Priority inheritance for the Pathfinder example

NO reset by watchdog timer

priority(J_1) > priority(J_2) > priority(J_3)

J_3 inherits priority of J_1

J_1 blocked
Priority inversion on Mars

- Priority inheritance also solved the Mars Pathfinder problem:
  - the VxWorks operating system used in the pathfinder implements a flag for the calls to mutual exclusion primitives.
  - This flag allows priority inheritance to be set to “on”.
  - When the software was shipped, it was set to “off”.

The problem on Mars was corrected by using the debugging facilities of VxWorks to change the flag to “on”, while the Pathfinder was already on the Mars [Jones, 1997].
Schedulability check

Let $B_i$ be the maximum blocking time due to lower-priority jobs that a job $J_i$ may experience.

\[ \forall \ i: R_i^{(0)} = C_i \]

repeat
\[ \forall \ i: R_i^{(j+1)} = C_i + B_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(j)}}{T_k} \right\rceil \cdot C_k \]
until (\exists \ i \text{ with } R_i^{(j+1)} > D_i) or (\forall \ i \ R_i^{(j+1)} = R_i^{(j)});

if (\forall \ i \ R_i^{(j+1)} = R_i^{(j)}) then

report("RM schedulable");
Blocking Time Computation

- Precise algorithm based on exhaustive search: exponential cost
- Here: approximative solution
- Assumption: no nested critical sections

**Lemma**: Transitive priority inheritance can only occur in the presence of nested critical sections.
Blocking Time

priority ceiling $C(S) =$ priority of the highest-priority job that can lock $S$

**Theorem:** In the absence of nested critical sections, a critical section of job $J$ guarded by semaphore $S$ can only block job $J'$ if $\text{priority}(J) < \text{priority}(J') \leq C(S)$. 
Blocking Time

- $D_{j,k}$: duration of longest critical section of task $\tau_j$, guarded by semaphore $S_k$

- Blocking Time
  
  - $B_i \leq \sum_{j=i+1}^{n} \max_k[D_{j,k} : C(S_k) \geq P_i]$
  - $B_i \leq \sum_{k=1}^{m} \max_{j>i}[D_{j,k} : C(S_k) \geq P_i]$

  where the task set consists of $n$ periodic tasks that use $m$ distinct semaphores.
Problem: Chained Blocking
Problem: Deadlock

\[
J_1: \text{wait}(S_a) \quad \text{wait}(S_b) \quad \text{signal}(S_b) \quad \text{signal}(S_a)
\]

\[
J_2: \text{wait}(S_b) \quad \text{wait}(S_a) \quad \text{signal}(S_a) \quad \text{signal}(S_b)
\]
Priority Ceiling Protocol

- Each semaphore S is assigned a **priority ceiling**: C(S)=priority of the highest-priority job that can lock S

- The processor is assigned to a ready job J with highest priority.

- To enter a critical section, J needs priority > C(S*), where S* is the currently locked semaphore with max C.
  → otherwise J „blocks on semaphore“ and priority of J is inherited by job J’ holding S*.

- When J’ exits critical section, its priority is updated to the highest priority of some job that is blocked by J’ (or to the nominal priority if no such job exists).
Example
Priority Ceiling Protocol

**Theorem (Sha/Rajkumar/Lehoczky):** Under the Priority Ceiling Protocol, a job can be blocked for at most the duration of one critical section.
Priority Ceiling Protocol

The Priority Ceiling Protocol prevents deadlocks.