Prof. Dr. Christian Steger
Prof. Dr. Reinhard Wilhelm

Sebastian Altmeyer, M.Sc.
Michael Gerke, B.Sc.
Dipl.-Inf. Hans-Jörg Peter

## Embedded Systems 2010/2011 - Assignment Sheet 3

Due: Tuesday, $16^{\text {th }}$ November 2010, before the lecture (i.e., 10:10)
Please indicate your name, matr. number, email address, and which tutorial you are planning to attend on your submission. We encourage you to collaborate in groups of up to three students. Only one submission per group is necessary. However, in the tutorials every group member must be capable to present each solution.

## Exercise 1: Esterel Scade

Consider the SyncChart diagram of Figure 1. The signals timer and input are global signals, all other signals are local.
(a) Give all legal configurations for the given SyncChart model.
(b) Which of these legal configurations are stable?
(c) Compute the reaction for the configuration $\{$ CHKM, CHKS, Off , Passive $\}$ for the event timer+. Use the algorithm presented in the lecture and give all substeps.


Figure 1: SyncChart for Exercise 1.

## Exercise 2: Lustre

In this exercise, your task is to provide a Lustre implementation of the Taylor series of the sine function for a given constant $x \in \mathbb{R}$ :

$$
\sin (x)=\sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2 k+1)!} x^{2 k+1}=x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\frac{x^{7}}{7!}+\ldots
$$

Provide a Lustre implementation of a node that produces a sequence of numbers converging to $\sin (x)$ for a given constant $x$. In your solution, you can introduce auxiliary nodes but you are only allowed to use the temporal operators pre, ->, when, and current, as well as, addition, subtraction, multiplication, and division.

## Exercise 3: Arithmetic Operations with Petri Nets

For each subtask of this exercise, construct a Petri net that comprises two input places $a$ and $b$, and one output place $z$ (additionally to the internal places that you might add to do the actual computation). The input of the arithmetic operation is specified in terms of the initial markings $M_{0}(a)$ and $M_{0}(b)$. The transitions in the net are fired until some final marking is reached, where no firing is possible. Recall that, due to the non-determinism in the order of the transition firings of a Petri net, there can be multiple final markings

$$
M_{\infty}^{0}, M_{\infty}^{1}, M_{\infty}^{2}, M_{\infty}^{3}, \ldots .
$$

You are only allowed to use constants (not the actual input values) to specify the initial markings of the internal places and $z$. In your submission, please use the graphical notation for the Petri nets.
(a) Construct a Petri net such that for all reachable final markings $M_{\infty}^{i}, i \geq 0$, $M_{\infty}^{i}(z)=M_{0}(a)+M_{0}(b)$.
(b) Construct a Petri net such that for all reachable final markings $M_{\infty}^{i}, i \geq 0$, $M_{\infty}^{i}(z)=\max \left(0, M_{0}(a)-M_{0}(b)\right)$.
(c) Construct a Petri net such that $\max _{i \geq 0} M_{\infty}^{i}(z)=\frac{M_{0}(a)}{2} \cdot\left(M_{0}(a)+1\right)$.

Note that $M_{0}(b)$ can be ignored here.


Figure 2: Petri net for Exercise 2 modeling a producer/consumer pattern.
Consider the Petri net in Figure 2 with places $A, \ldots, H$ and transitions $t_{1}, \ldots t_{6}$. Assume an initial marking $M_{0}$ with $M_{0}(A)=1, M_{0}(H)=3, M_{0}(D)=1$, and $M_{0}(p)=0$ for every other place $p$.
(a) Compute the incidence matrix for the Petri net.
(b) Use the incidence matrix to deduce all place invariants. Write down your intermediate steps.
(c) Is the net bounded? Justify your answer.

