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Problem Set 10
discussion session: 15th of January 2013

Automata, Games, and Verification

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1. **Arena-preserving game conversions** (Group G05, discussion session 12:00 with Felix Klein)

   Let an arena $\mathcal{A} = (V_0, V_1, E)$ and a position $v_{in} \in V_0$ be given. Consider all combinations (a, b) of winning conditions from the list below. For which of the combinations does it hold that if you are given an a-type winning condition $\mathcal{F}$, you can always convert it to a b-type winning condition $\mathcal{F}'$ such that the winning strategies for the two players from v_{in} in $(\mathcal{A}, \mathcal{F})$ are the same as from v_{in} in $(\mathcal{A}, \mathcal{F}')$?

   List of winning condition types:

   - Reachability games
   - Büchi games
   - Parity games
   - Rabin games
   - Muller games

2. **Fair Simulation** (Group G09, discussion session 12:00 with Hazem Torfah)

   Simulation is often used as an efficient method to establish language containment between automata. *Fair simulation*, the commonly used type of simulation for Büchi automata, can be described as a game:

   Given two Büchi automata $\mathcal{A} = (S_\mathcal{A}, \{i_\mathcal{A}\}, T_\mathcal{A}, F_\mathcal{A})$, $\mathcal{B} = (S_\mathcal{B}, \{i_\mathcal{B}\}, T_\mathcal{B}, F_\mathcal{B})$, the automaton $\mathcal{B}$ *simulates* the automaton $\mathcal{A}$ if player "Duplicator" wins the following game between Duplicator and a second player "Spoiler." The game is played in rounds, as follows: At the start, round 0, two pebbles, a and b, are placed on $i_\mathcal{A}$ and $i_\mathcal{B}$, respectively. Assume that, at the beginning of round n, pebble a is on state $q_n \in S_\mathcal{A}$ and pebble b is on state $q'_n \in S_\mathcal{B}$. Then:

   a) Spoiler chooses a transition $(q_n, l, q_{n+1}) \in T_\mathcal{A}$ for some letter $l \in \Sigma$.
   b) Duplicator, responding, must chose a transition $(q'_n, l, q'_{n+1}) \in T_\mathcal{B}$. If no l-transition exists from q'_n, Spoiler wins the game at this point.

   If a play is infinite, then the winner is determined according to the following rule: Duplicator wins if there are infinitely many j such that $q'_j \in F_\mathcal{B}$ or there are only finitely many j such that $q_j \in F_\mathcal{A}$.

   a) For two given Büchi automata $\mathcal{A}, \mathcal{B}$, reformulate this game as a parity game such that $\mathcal{B}$ simulates $\mathcal{A}$ iff player 1 wins the parity game (from some designated starting position).
   b) Show that fair simulation is a conservative test for language containment. I.e., show that if $\mathcal{B}$ simulates $\mathcal{A}$, then $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$, and give an example where $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$, but $\mathcal{B}$ does not simulate $\mathcal{A}$.

3. **Tree Automata** (Group G14, discussion session 12:20 with Hazem Torfah)

   a) Give a Büchi tree automaton for the language
   $$L_1 = \{v \in T_{\{a,b\}} \mid \text{ there is a branch in } v \text{ with infinitely many } a\}$$
   b) Give a co-Büchi tree automaton for the language
   $$L_2 = \{v \in T_{\{a,b,c\}} \mid \begin{array}{l}\text{each branch in } v \text{ has at least one } a\\ \text{and the entire tree has at most one } b\}\end{array}$$

c) Give a Muller tree automaton for the language

$$L_3 = \{v \in T_{\{a,b\}} \mid \text{ each branch in } v \text{ has only finitely many } a\}$$

4. **Update Games** (Challenge question)

   Reconsider the class of update games that was defined on the previous problem sheet.

   a) Prove the following:

      If $\mathcal{G}$ is an update game with a forced cycle of length ≥ 4, then we can construct a new game $\mathcal{G}'$ with fewer nodes, such that $\mathcal{G}$ is an update network iff $\mathcal{G}'$ is an update network.

   b) Using this result, provide a decision procedure that determines whether an update game is an update network.