Automata, Games & Verification

#9

Theorem 1. *Every QPTL-definable language is S1S-definable.*

Theorem 2. *Every S1S-definable language is Büchi-recognizable.*

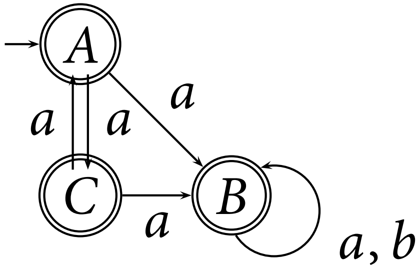
Theorem 3. *A language is WS1S-definable iff it is S1S-definable.*

Hence:

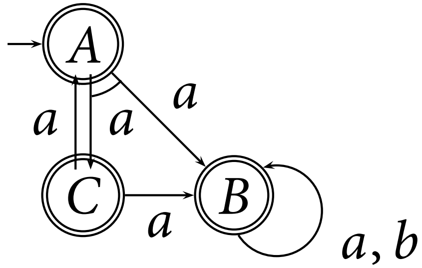
$$\text{LTL} \subsetneq \text{QPTL} \subseteq \text{(W)S1S} \subseteq \text{Büchi} \subseteq \text{QPTL}.$$

Alternating Automata

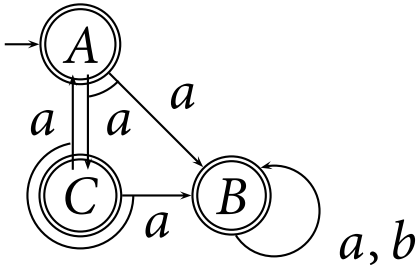
- nondeterministic automaton,
  $L = a(a+b)^\omega$:



- alternating automaton,
  $L = aa(a+b)^\omega$



- universal automaton, $L = a^\omega$:



Definition 1. *An alternating Büchi automaton is a tuple* $\mathcal{A} = (S, s_0, \delta, F)$*, where:*

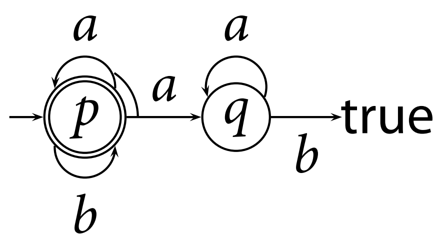
- S *is a finite set of states,*
- $s_0 \in S$ *is the initial state,*
- $F \subseteq S$ *is the set of accepting states, and*
- $\delta : S \times \Sigma \to \mathbb{B}^+(S)$ *is the transition function.*

Definition 2. *A run of an alternating automaton on a word* $\alpha \in \Sigma^\omega$ *is an* S*-labeled tree* $\langle T, r \rangle$ *with the following properties:*

- $r(\varepsilon) = s_0$ *and*
- *for all* $n \in T$,
  if $r(n) = s$*, then* $\{r(n') \mid n' \in \mathit{children}(n)\}$ *satisfies* $\delta(s, \alpha(|n|))$.

Example:

$L = (\{a, b\}^* \, b)^\omega$



$S = \{p, q\}$
$F = \{p\}$;
$\delta(p, a) = p \wedge q$; $\delta(p, b) = p$; $\delta(q, a) = q$; $\delta(q, b) =$ true

example word $w = (aab)^\omega$ has the following run: