Automata, Games & Verification

Summary #5

Complementation of Büchi Automata

Lemma 1.

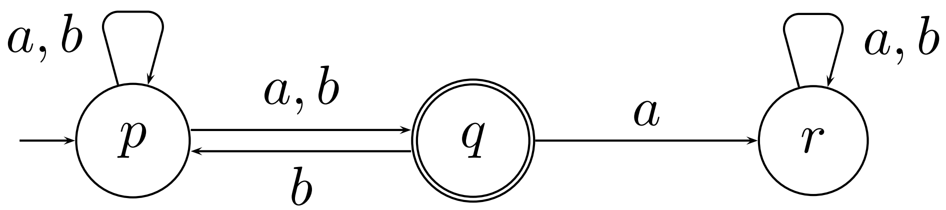
If there exists an odd ranking for G, then $\mathcal{A}$ does not accept α.

Let G' be a subgraph of G. We call a vertex $\langle s, l \rangle$

- safe in G' if for all vertices $\langle s', l' \rangle$ reachable from $\langle s, l \rangle$, $s' \notin F$, and
- endangered in G' if only finitely many vertices are reachable.

We define an infinite sequence $G_0 \supseteq G_1 \supseteq G_2 \supseteq \ldots$ of DAGs inductively as follows:

- $G_0 = G$
- $G_{2i+1} = G_{2i} \setminus \{\langle s, l \rangle \mid \langle s, l \rangle$ is endangered in $G_{2i}\}$
- $G_{2i+2} = G_{2i+1} \setminus \{\langle s, l \rangle \mid \langle s, l \rangle$ is safe in $G_{2i}\}$.


a, b
a, b
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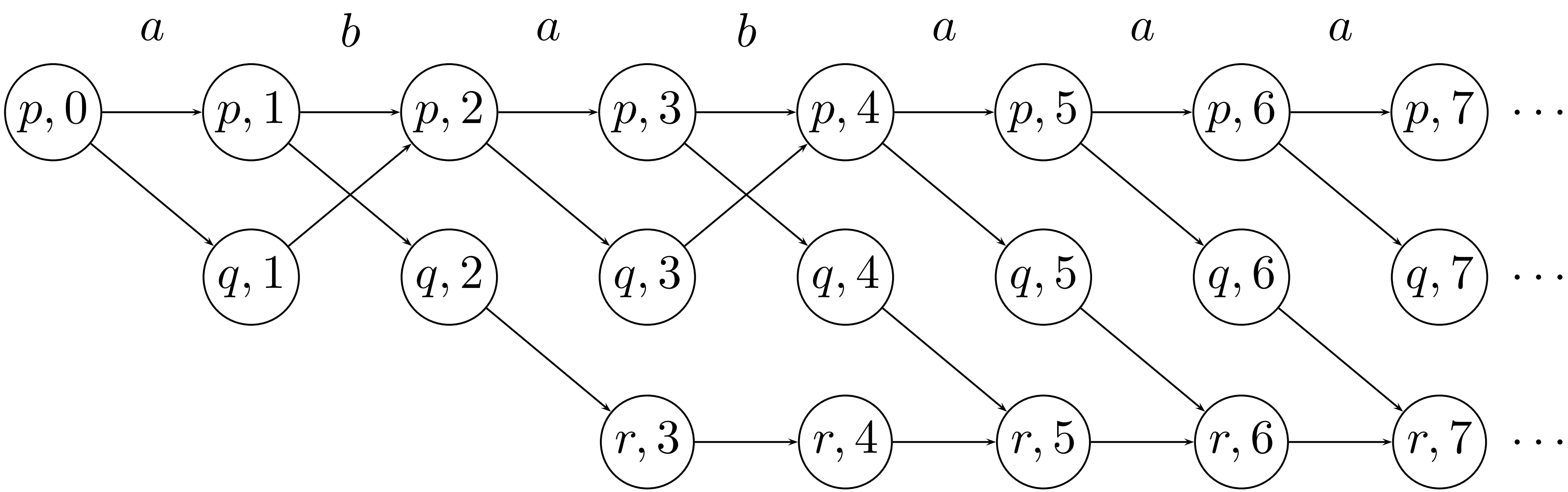
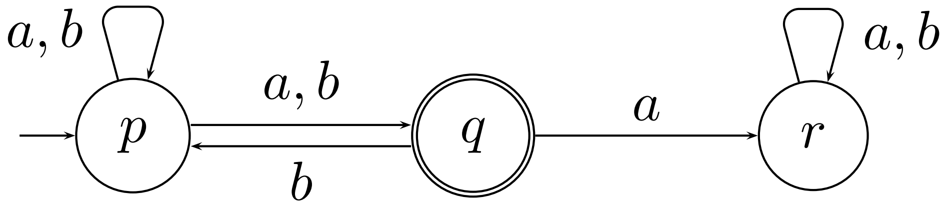
a
b
p
q
r


a b a b a a a
$p, 0$
$p, 1$
$p, 2$
$p, 3$
$p, 4$
$p, 5$
$p, 6$
$p, 7$
$\cdots$
$q, 1$
$q, 2$
$q, 3$
$q, 4$
$q, 5$
$q, 6$
$q, 7$
$\cdots$
$r, 3$
$r, 4$
$r, 5$
$r, 6$
$r, 7$
$\cdots$

$$G = G_0 = G_1$$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a, b
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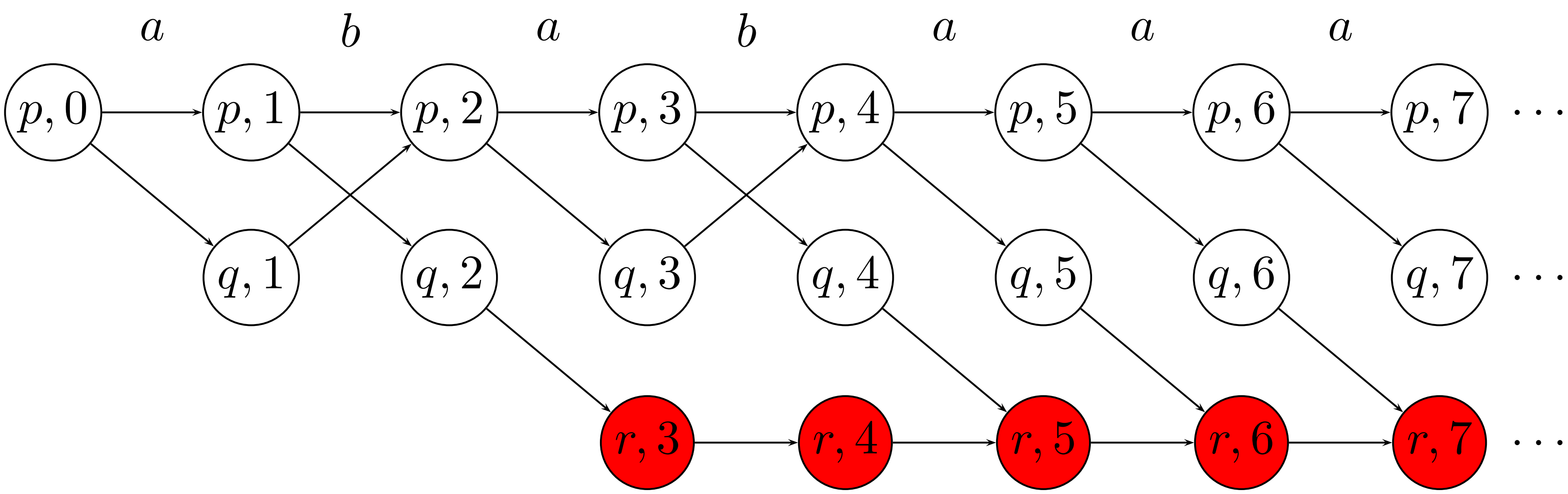
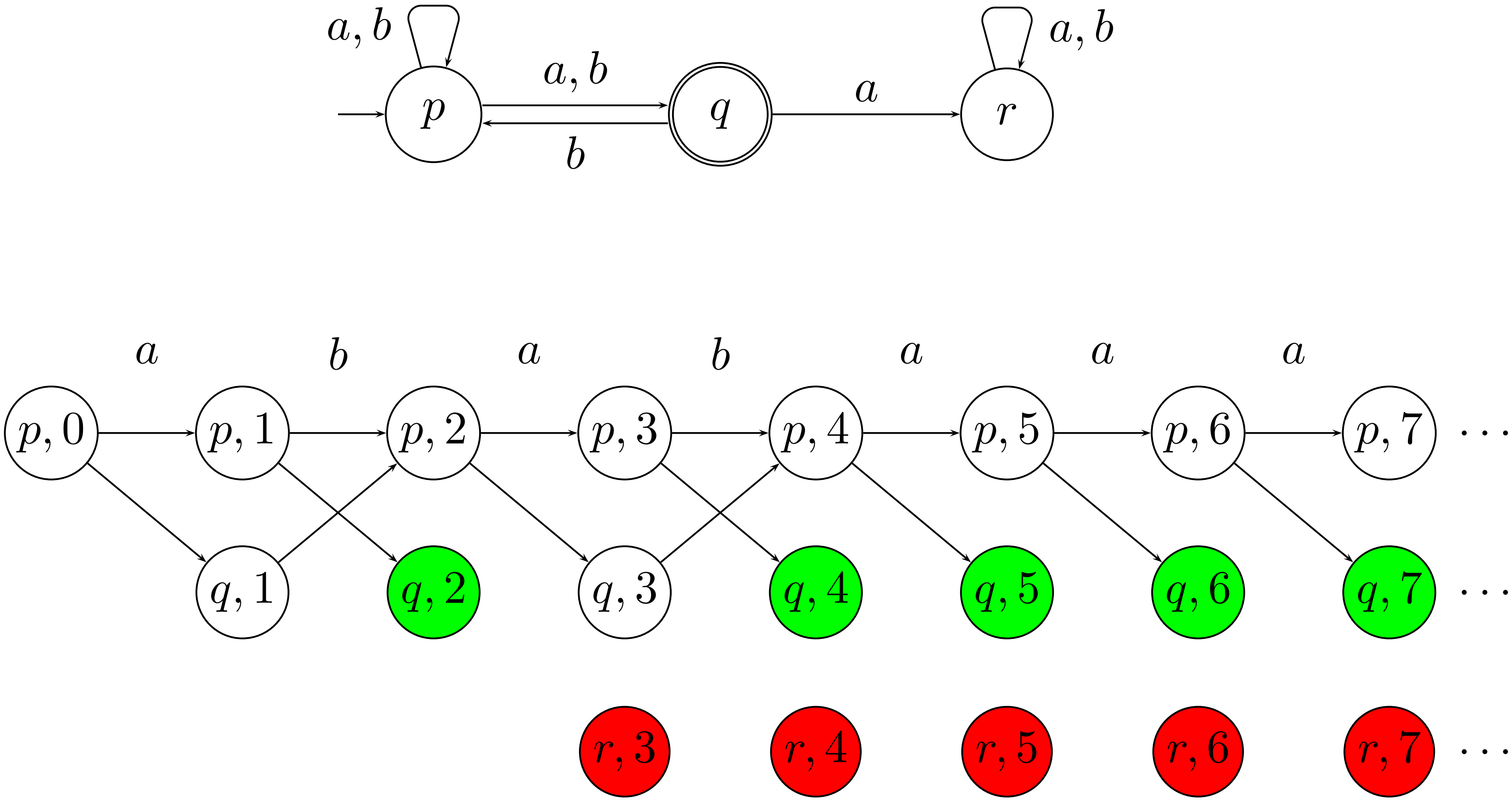

a b a b a a a
p, 0
p, 1
p, 2
p, 3
p, 4
p, 5
p, 6
p, 7
q, 1
q, 2
q, 3
q, 4
q, 5
q, 6
q, 7
r, 3
r, 4
r, 5
r, 6
r, 7
. . .

G_1


a, b
a, b
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p, 0
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p, 3
p, 4
p, 5
p, 6
p, 7
q, 1
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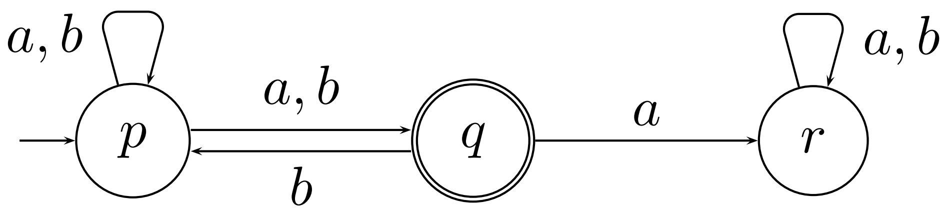
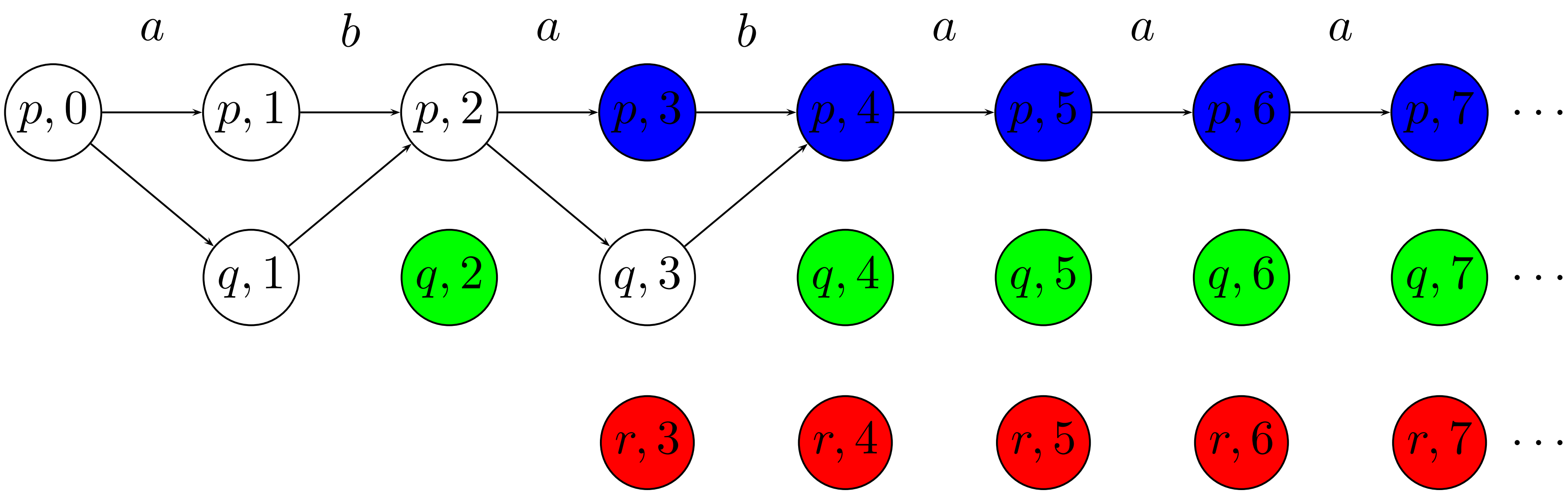
r, 5
r, 6
r, 7

G_2


a, b
a, b
p
q
a
r
a, b
b


a
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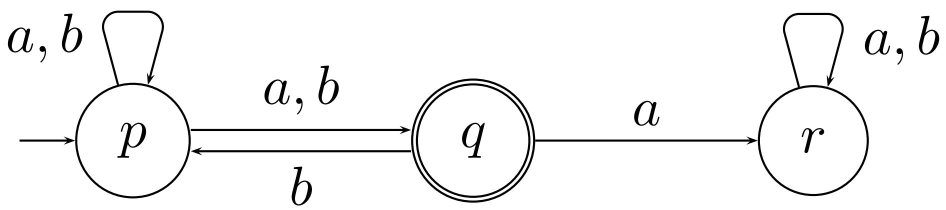
p, 0
p, 1
p, 2
p, 3
p, 4
p, 5
p, 6
p, 7
· · ·
q, 1
q, 2
q, 3
q, 4
q, 5
q, 6
q, 7
· · ·
r, 3
r, 4
r, 5
r, 6
r, 7
· · ·

G_3


a, b
a, b
p
q
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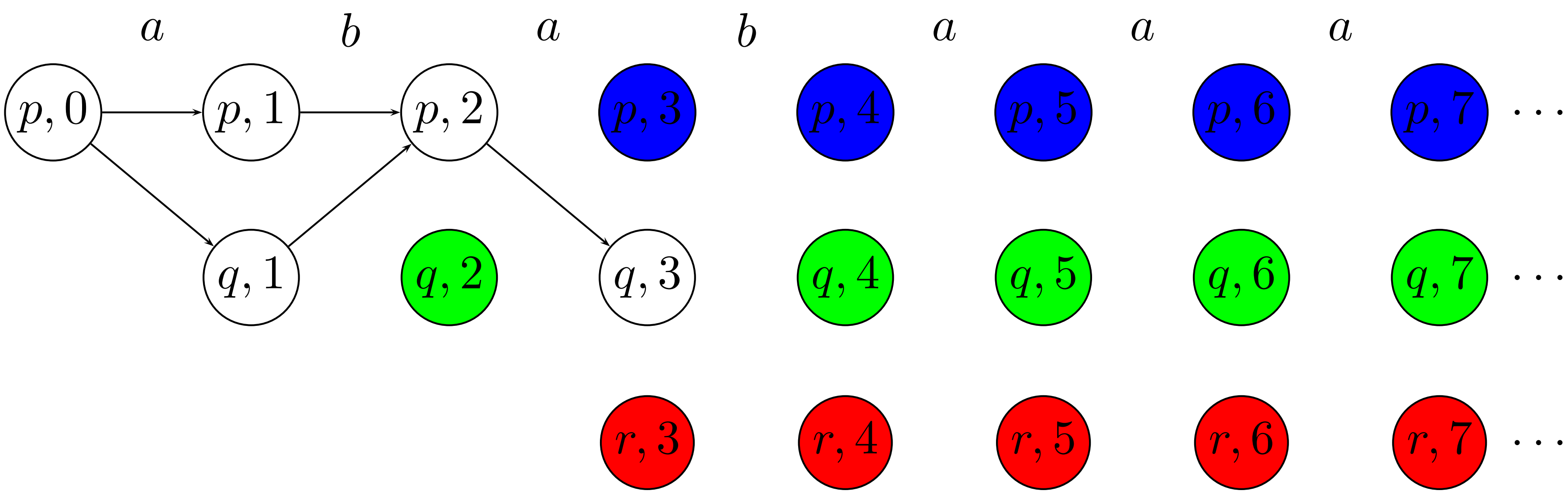
a
r
a, b


a b a b a a a
p, 0
p, 1
p, 2
p, 3
p, 4
p, 5
p, 6
p, 7
q, 1
q, 2
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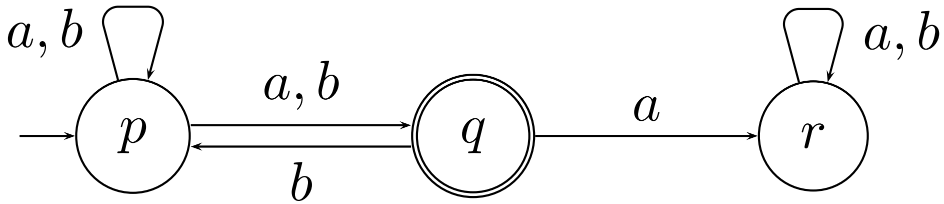
q, 4
q, 5
q, 6
q, 7
r, 3
r, 4
r, 5
r, 6
r, 7

G_4


a, b
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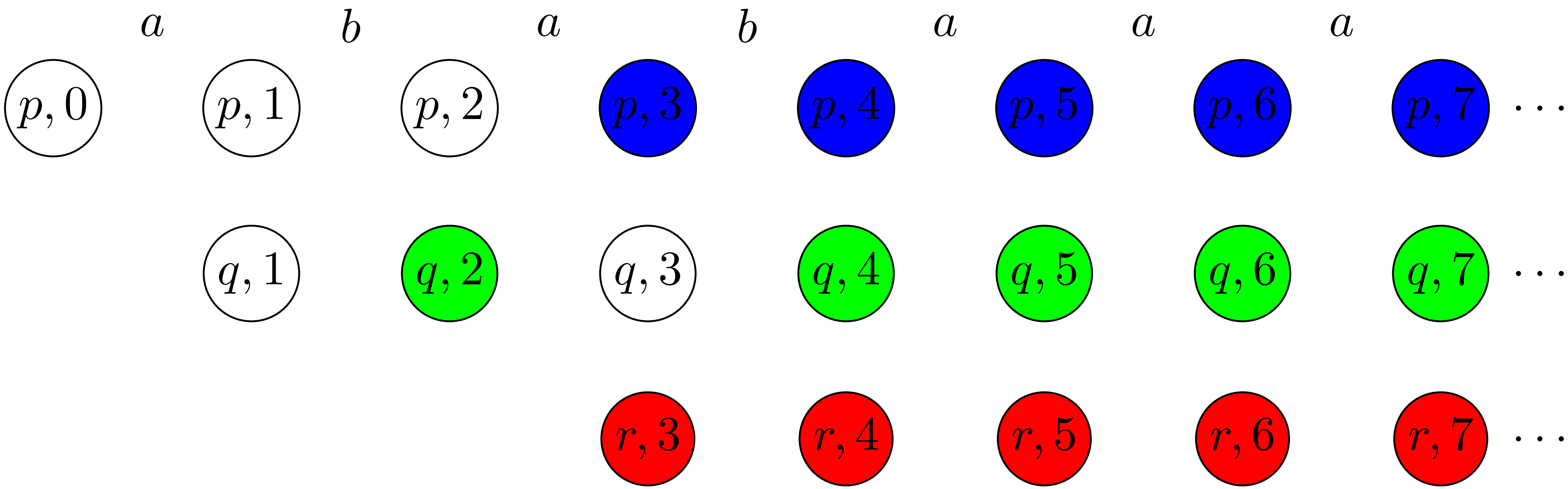
p
q
b
a
r
a, b


a
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p, 0
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q, 1
q, 2
q, 3
q, 4
q, 5
q, 6
q, 7
. . .
r, 3
r, 4
r, 5
r, 6
r, 7
. . .

G_5

Lemma 2.

If $\mathcal{A}$ does not accept α, then the following holds:
For every $i \geqslant 0$ there exists an l_i such that
for all $j \geqslant l_i$ at most $|S| - i$ vertices of the form $\langle \_, j \rangle$ are in G_{2i}.

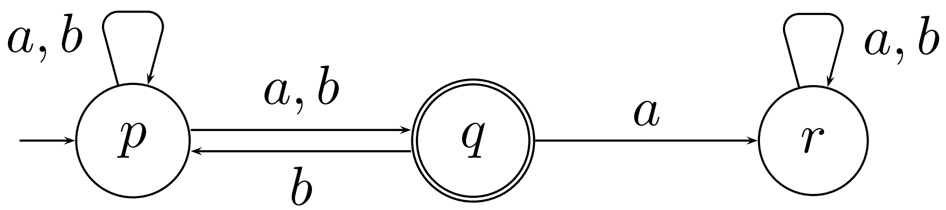
Proof by induction on i.

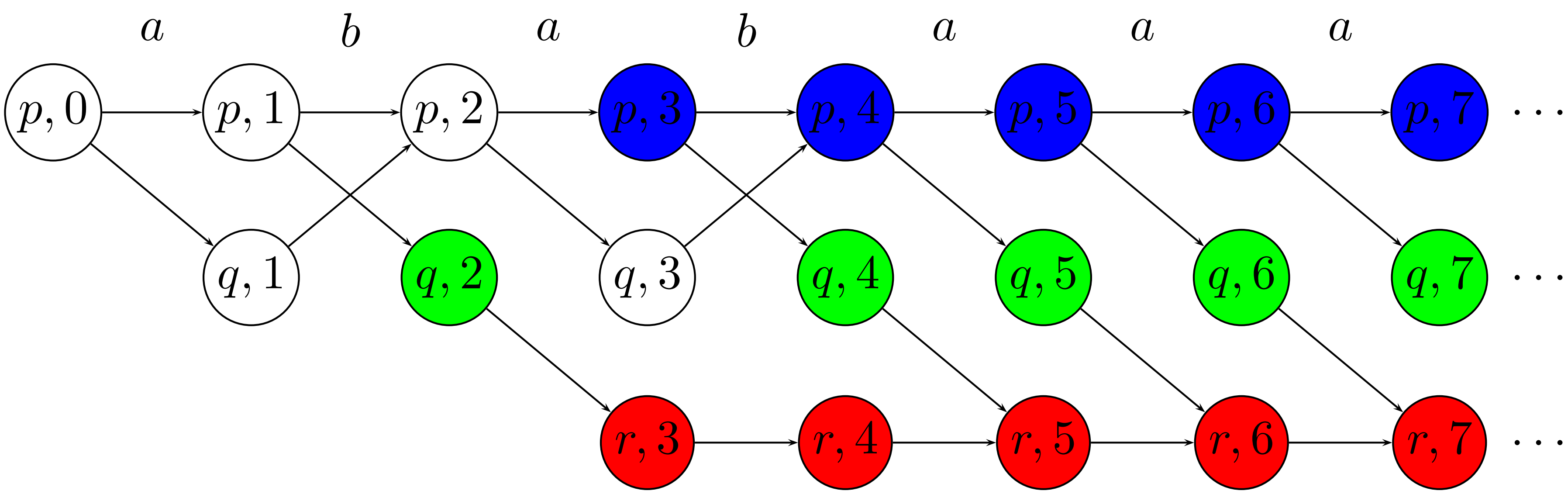
Lemma 3.

If $\mathcal{A}$ does not accept α, then there exists an odd ranking for G.

We defined the ranking f where:

- $f(\langle s, l \rangle)$ = $2i$ if $\langle s, l \rangle$ is endangered in G_{2i}
- $f(\langle s, l \rangle)$ = $2i + 1$ if $\langle s, l \rangle$ is safe in G_{2i+1}

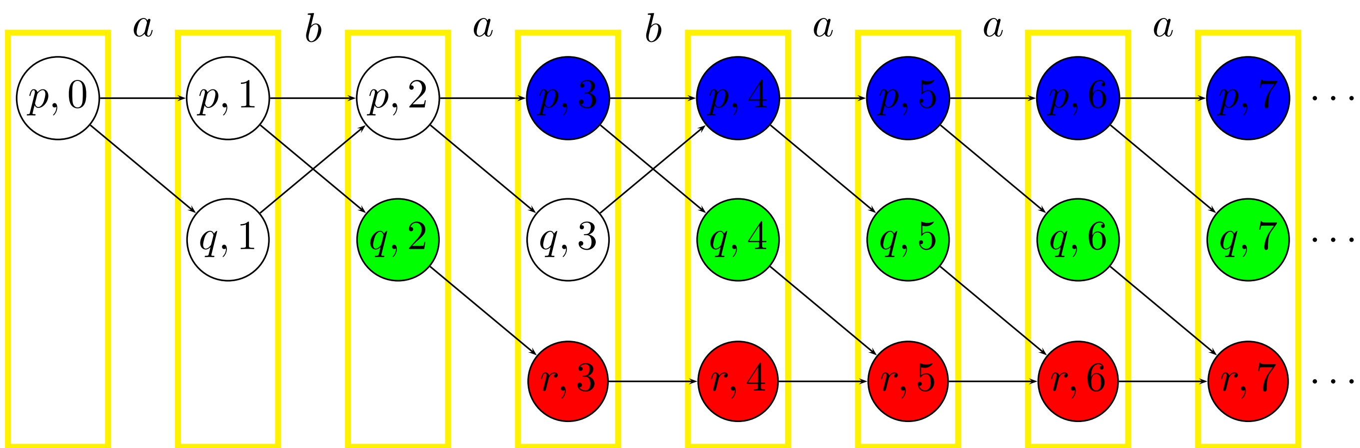




rank 1 — rank 2 — rank 3 — rank 4

Complementation

Theorem 1. *For each Büchi automaton $\mathcal{A}$ there exists a Büchi automaton $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$.*



Determinizing

Theorem 2. *The language* $(a \cup b)^* b^\omega$ *is not recognizable by a deterministic Büchi automaton.*