Automata, Games & Verification

Summary #7

Today at 2:15pm in SR 016

Seminar "Games, Synthesis, and Robotics"

Efficient On-the-fly Algorithms for the Analysis of Timed Games

QPTL

Syntax: LTL formula | $\varphi \wedge \varphi$ | $\neg\varphi$ | $\exists p.\ \varphi$

Semantics:

$\alpha, i \models \exists q.\varphi$ iff there is an α' with
$\alpha'(j) \cap (AP \setminus \{q\}) = \alpha(j) \cap (AP \setminus \{q\})$ for all $j \in \omega$,
s.t. $\alpha', i \models \varphi$.

(W)S1S

Syntax:

- Terms $t ::= 0 \mid x \mid S(t)$
- Formulas $\varphi ::= t \in X \mid t_1 = t_2 \mid \neg\varphi \mid \varphi_0 \vee \varphi_1 \mid \exists x.\varphi \mid \exists X.\varphi$

Semantics (S1S):

$\sigma_1, \sigma_2 \models \exists X.\varphi$ iff there is a $A \subseteq \omega$ s.t.

$$\sigma_2'(X) = \begin{cases} \sigma_2(X) \text{ if } X \neq X_i \\ A \text{ otherwise} \end{cases} \qquad \text{and} \qquad \sigma_1, \sigma_2' \models \varphi.$$

Semantics (WS1S):

$\sigma_1, \sigma_2 \models \exists X.\varphi$ iff there is a **finite** $A \subseteq \omega$ s.t. $\ldots$

Theorem 1. *Every QPTL-definable language is S1S-definable.*

Theorem 2. *Every S1S-definable language is Büchi-recognizable.*

Theorem 3. *A language is WS1S-definable iff it is S1S-definable.*

Hence:

$$\text{LTL} \subsetneq \text{QPTL} \subseteq \text{(W)S1S} \subseteq \text{Büchi} \subseteq \text{QPTL}.$$

Examples: Problem Set 7, Question 1

Decide for each of the languages over $2^{\{p,q\}}$ described below if they can be defined in S1S and/or LTL. Justify your answer in each case by either providing a formula or an argument why the language is not definable.

(a) $L_1 = \{\alpha \mid p \in \alpha(0), p \notin \alpha(i) \text{ for all } i \geqslant 1\}$

- S1S: $\forall x.\ x \in P \leftrightarrow x = 0$
- LTL: $p \wedge \bigcirc\Box\neg p$

Examples: Problem Set 7, Question 1

(b) $L_2 = \{\alpha \mid p \in \alpha(i)$ for exactly two different $i \in \omega\}$

- S1S: $\exists x.\exists y.\, x \neq y \wedge \forall z.\ z \in P \leftrightarrow (x = z \vee y = z)$

- LTL: $\neg p\,\mathcal{U}\left(p \wedge \bigcirc(\neg p\,\mathcal{U}\,(p \wedge \bigcirc\Box\neg p))\right)$

Examples: Problem Set 7, Question 1

(c) $L_3 = \{\alpha \mid \ |\{i \in \omega \,|\, p \in \alpha(i)\}|$ is finite and even$\}$

- S1S: $\varphi \ = \ \exists O \ \exists E \ \Big(\forall x.\ x \in P \leftrightarrow x \in O \vee x \in E$

$$\begin{aligned}
&\wedge \forall x. \neg(x \in O) \vee \neg(x \in E) \\
&\wedge \exists y. \forall x.\ x \in E \leftrightarrow x < y \\
&\wedge \exists y. y \in O \ \wedge \ (\forall x. x \in E \rightarrow x > y) \\
&\wedge \exists y. y \in E \ \wedge \ (\forall x. x \in O \rightarrow x < y) \\
&\wedge \forall x. \forall y. x \in O \wedge y \in O \wedge x < y \ \rightarrow \exists z. x < z < y \wedge z \in E \\
&\wedge \forall x. \forall y. x \in E \wedge y \in E \wedge x < y \ \rightarrow \exists z. x < z < y \wedge z \in O \Big)
\end{aligned}$$

- LTL: There is no translation, since every LTL definable language is non-counting.

Examples: Problem Set 7, Question 1

(d) $L_4 = \{\alpha \mid \ |\{i \in \omega \,|\, p \in \alpha(i)\}|$
and $|\{i \in \omega \,|\, q \in \alpha(i)\}|$ are finite and equal$\}$

The language is not ω-regular, and hence not expressible in S1S or LTL.