Automata, Games & Verification

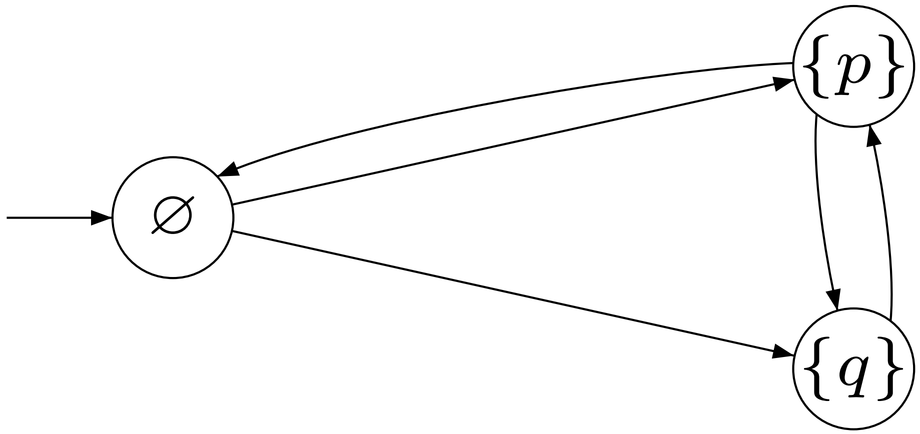
Summary #13

Today at 2:15pm in SR 016

Seminar “Games, Synthesis, and Robotics”

Alternating-Time Temporal Logic

Kripke structures



Definition 1. *Let AP be a set of atomic propositions. A Kripke structure over AP is a tuple* $\mathcal{M} = (S, R, L)$

- S *: a set of states*
- $R \subseteq S \times S$ *: a transition relation*
- $L : S \rightarrow 2^{AP}$ *: labels each state with the set of atomic propositions that are assured to be true in* S

Computation Tree Logic

CTL* Syntax (f, g - state formulas, φ, ψ - path formulas):

- State formulas f:

$$f \ ::= \ AP \mid \neg f \mid f \vee g \mid A\varphi \mid E\varphi$$

- Path formulas φ:

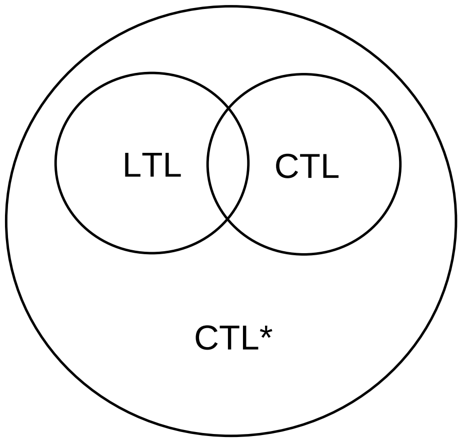
$$\varphi ::= \ f \mid \neg\varphi \mid \varphi \vee \psi \mid G\varphi \mid F\varphi \mid \varphi U \psi \mid X\varphi$$

CTL* Semantics ($\mathcal{M}$ - Kripke structure, s - state, π^i - suffix of π starting at i):

- $\mathcal{M}, s \models p$ iff $p \in L(s)$ for $p \in AP$
- $\mathcal{M}, s \models \neg f$ iff $\mathcal{M}, s \not\models f$
- $\mathcal{M}, s \models$ Eφ iff there is a path π from s such that $\mathcal{M}, \pi \models \varphi$
- $\mathcal{M}, s \models$ Aφ iff for every path π from s such that $\mathcal{M}, \pi \models \varphi$

- $\mathcal{M}, \pi \models f$ iff $\mathcal{M}, s \models f$ where $\pi = s\pi^1$
- $\mathcal{M}, \pi \models \neg\varphi$ iff $\mathcal{M}, \pi \not\models \varphi$
- $\mathcal{M}, \pi \models \varphi \vee \psi$ iff $\mathcal{M}, \pi \models \varphi$ or $\mathcal{M}, \pi \models \psi$
- $\mathcal{M}, \pi \models \varphi\, \mathcal{U}\, \psi$ iff there exists i such that for every $j < i$
  $\mathcal{M}, \pi^j \models \varphi$ and $\mathcal{M}, \pi^i \models \psi$
- $\mathcal{M}, \pi \models$ Xφ iff $\mathcal{M}, \pi^1 \models \varphi$

LTL. Special case of CTL* formulas: A φ, where φ is a path formula with only atomic propositions as state subformulas.
CTL. Special case of CTL* formulas where each temporal operator must immediately be preceeded by a path quantifier.



Theorem 1. *For every CTL* formula* Φ, *the following are equivalent:*

1. *there is an LTL formula* $A\varphi$ *that is equivalent to* Φ
2. Φ *is equivalent to* $A(remove_{E,A}(\Phi))$, *where* $remove_{E,A}(\Phi)$ *is obtained from* Φ *by deleting all path quantifiers.*

The Modal μ-calculus

Syntax: given a set of atomic propositions AP, the set of formulas is defined inductively as follows (where φ and ψ are formulas)

- $\bot, \top$
- $p, \neg p$ for every $p \in AP$
- $\varphi \wedge \psi$, $\varphi \vee \psi$
- $\Box\varphi$, $\Diamond\varphi$
- $\mu p\ \varphi$, $\nu p\ \varphi$, where $p \in AP$ and p only occurs positively in φ.

Semantics: Formulas are interpreted as sets of states.

- $\|\bot\|_{\mathcal{M}} = \emptyset$
- $\|\top\|_{\mathcal{M}} = S$
- $\|p\|_{\mathcal{M}} = \{s | p \in L(s)\}$
- $\|\neg p\|_{\mathcal{M}} = \{s | p \notin L(s)\}$
- $\|\varphi \vee \psi\|_{\mathcal{M}} = \|\varphi\|_{\mathcal{M}} \cup \|\psi\|_{\mathcal{M}}$, $\|\varphi \wedge \psi\|_{\mathcal{M}} = \|\varphi\|_{\mathcal{M}} \cap \|\psi\|_{\mathcal{M}}$
- $\|\Box\varphi\|_{\mathcal{M}} = \{s | \forall t.(s,t) \in R \rightarrow t \in \|\varphi\|_{\mathcal{M}}\}$
- $\|\Diamond\varphi\|_{\mathcal{M}} = \{s | \exists t.(s,t) \in R \wedge t \in \|\varphi\|_{\mathcal{M}}\}$
- $\|\mu p.\varphi\|_{\mathcal{M}} = \bigcap\{S' \subseteq S \mid \|\varphi\|_{\mathcal{M}[p \mapsto S]} \subseteq S'\}$
- $\|\nu p.\varphi\|_{\mathcal{M}} = \bigcup\{S' \subseteq S \mid \|\varphi\|_{\mathcal{M}[p \mapsto S]} \supseteq S'\}$
- $(\mathcal{M}, s) \models \varphi$ iff $s \in \|\varphi\|_{\mathcal{M}}$