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Problem Set 8
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Automata, Games, and Verification

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1. **LTL-to-Alternating-Büchi** (tutorial A: group G09, tutorial B: group G12)

   Give an alternating Büchi automaton $\mathcal{A}$ and a nondeterministic Büchi automaton $\mathcal{A}'$ such that
   $$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}') = models((\mathsf{F}p)\ \mathcal{W}\ (\mathsf{G}q)).$$
   Use the construction from the lecture to obtain $\mathcal{A}$.

2. **Complete Alternating Büchi Automata** (tutorial A: group G11, tutorial B: group G02)

   An alternating automaton is called *complete* iff neither *true* nor *false* are in the mapping of δ (run trees of complete alternating automata have only infinite branches and every input word has a run tree).

   Prove or give a counter-example to the following statement:
   Every language that is recognized by an alternating Büchi automaton is recognized by a complete alternating Büchi automaton.

3. **Alternating vs. Deterministic Automata** (challenge problem)

   Consider the following family of languages L_n:
   $$L_n = \{v_1 \# u\, v_2 \$ u\, \beta \mid \begin{array}{l} v_1 \in \{0,1,\#\}^* \\ v_2 \in \{0,1,\#\}^* \\ u \in \{0,1\}^n \\ \beta \in \{0,1,\#,\$\}^\omega \end{array} \}.$$

   a) Construct a family $\mathcal{A}_n$ of alternating Büchi automata with $\mathcal{L}(\mathcal{A}_n) = L_n$ such that each automaton $\mathcal{A}_n$ has only $O(n)$ states.

   b) Show that any deterministic Muller automaton that recognizes L_n has at least 2^{2^n} states.