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Problem Set 12
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Automata, Games, and Verification

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1. **CTL$^+$** (tutorial A: group G09, tutorial B: group G04)

   Consider the following fragment, called CTL$^+$, of CTL*, which extends CTL by allowing Boolean operators in path formulas:

   - State formulas f:
   $$f ::= \; AP \mid \neg f \mid f \vee g \mid A\varphi \mid E\varphi$$
   - Path formulas φ:
   $$\varphi ::= \; \neg\varphi \mid \varphi \vee \psi \mid Gf \mid Ff \mid f\,U g \mid Xf$$

   (Note: CTL* extends CTL$^+$ by allowing to use state formulas f as one more alternative in the definition of path formulas φ.)

   a) Provide, if they exist, equivalent CTL and LTL formulas for the CTL$^+$ formulas $A(F\,a \wedge G\,b)$ and $A(X\,a \wedge \neg(a\,U\,(G\,b)))$.

   b) Compare the expressive power of CTL$^+$ with the expressive power of CTL and LTL.

2. **Modal μ-calculus** (tutorial A: group G11, tutorial B: group G10)

   Let $\mathcal{M} = (S, R, L)$ be a Kripke structure, and $s \in S$ - some state in it.

   a) Give a μ-calculus formula φ such that the following holds: $s \in ||\varphi||_{\mathcal{M}}$ iff all paths in $\mathcal{M}$ starting in s are finite.

   b) Let ψ be an arbitrary μ-calculus formula. Give a μ-calculus formula φ with the following property: $s \in ||\varphi||_{\mathcal{M}}$ iff there exists a path π starting in s such that on this path the formula ψ holds true infinitely often, that is, the set $\{i \in \omega \mid \pi(i) \in ||\psi||_{\mathcal{M}}\}$ is infinite.

3. **CTL and S2S** (tutorial A: group G13, tutorial B: group G12)

   Describe a (recursive) procedure that translates a CTL formula ψ over some set of atomic propositions AP into an S2S formula ϕ using AP as the set of unbounded second-order quantifiers such that ϕ and ψ hold on the same set of 2^{AP}-labelled trees.