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Problem Set 11
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Automata, Games, and Verification

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1. **Tree Automata & S2S** (tutorial A: group G01, tutorial B: group G04)

   a) Give a Büchi tree automaton and an S2S formula for the language
   $L_1 = \{v \in T_{\{a,b\}} \mid$ there is a branch in v with infinitely many $a\}$

   b) Give a co-Büchi tree automaton and an S2S formula for the language
   $L_2 = \{v \in T_{\{a,b,c\}} \mid$ each branch in v has at least one a and the entire tree has at most one $b\}$

   c) Give a Muller tree automaton and an S2S formula for the language
   $L_3 = \{v \in T_{\{a,b\}} \mid$ each branch in v has only finitely many $a\}$

2. **Deterministic tree automata** (tutorial A: group G05, tutorial B: group G06)

   Compare the expressive power of deterministic and non-deterministic parity tree automata. We call a parity tree automaton $\mathcal{A} = (S, s_0, M, c)$ over the alphabet Σ deterministic if for every $s \in S$ and $x \in \Sigma$, there exists at most one pair $(s_1, s_2) \in S^2$ such that $(s, x, s_1, s_2) \in M$.

3. **Arena-preserving game conversions** (tutorial A: group G13, tutorial B: group G10)

   Let an arena $\mathcal{A} = (V_0, V_1, E)$ and a position $v_{in} \in V_0$ be given. Consider all combinations (a, b) of winning conditions from the list below. For which of the combinations does it hold that if you are given an a-type winning condition $\mathcal{F}$, you can always convert it to a b-type winning condition $\mathcal{F}'$ such that the winning strategies for the two players from v_{in} in $(\mathcal{A}, \mathcal{F})$ are the same as from v_{in} in $(\mathcal{A}, \mathcal{F}')$?

   List of winning condition types:

   - Reachability games
   - Büchi games
   - Parity games
   - Rabin games
   - Streett games
   - Muller games

   Hint: Use the fact that some of the winning conditions are special cases of others to reduce the number of cases to consider.

4. **Update Games** (Challenge question)

   Reconsider the class of update games that was defined on the previous problem sheet.

   a) Prove the following:

   If $\mathcal{G}$ is an update game with a forced cycle of length ≥ 4, then we can construct a new game $\mathcal{G}'$ with fewer nodes, such that $\mathcal{G}$ is an update network iff $\mathcal{G}'$ is an update network.

   b) Using this result, provide a decision procedure that determines whether an update game is an update network.