Automata, Games & Verification

Summary #9

Today at 4:15pm in SR 014

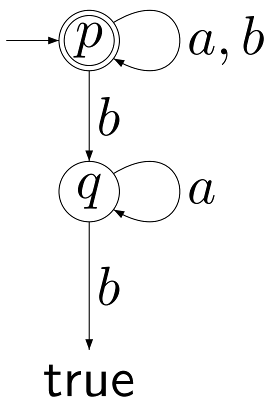
Games in Verification and Synthesis

Patrick Jungblut: *Timed Games*

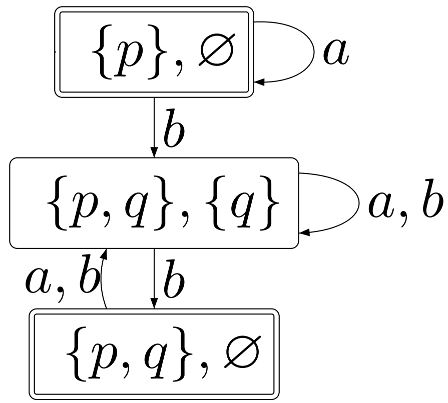
Christine Rizkallah: *Synthesis of Asynchronous Systems*

Corollary 1.

A language is ω-regular iff it is recognizable by an alternating Büchi automaton.



universal
automaton



nondeterministic
automaton

Definition 1. *Two nodes* $x_1, x_2 \in T$ *in a run tree* (T, r) *are* *similar* *if* $|x_1| = |x_2|$ *and* $r(x_1) = r(x_2)$.

Definition 2. *A run tree* (T, r) *is* *memoryless* *if for all similar nodes* x_1 *and* x_2 *and for all* $y \in D^*$ *we have that (*$x_1 \cdot y \in T$ *iff* $x_2 \cdot y \in T$*) and* $r(x_1 \cdot y) = r(x_2 \cdot y)$.

Theorem 1. *If an alternating Büchi Automaton* $\mathcal{A}$ *accepts a word* α*, then there exists a memoryless accepting run of* $\mathcal{A}$ *on* α.

Corollary 2.

Satisfiability of an LTL formula φ can be checked in time exponential in the length of φ.

Corollary 3.

Validity of an LTL formula φ can be checked in time exponential in the length of φ.

Theorem 2. [Tutorial] *Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be two alternating parity automata.*

1. *The language $\mathcal{L}(\mathcal{P}_1) \cup \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$ and $\mathcal{P}_2$.*

2. *The language $\mathcal{L}(\mathcal{P}_1) \cap \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$ and $\mathcal{P}_2$.*

3. *The language $\overline{\mathcal{L}(\mathcal{P}_1)}$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$.*

Which automata recognize the ω-regular languages?

	Büchi	co-Büchi	parity	Muller
deterministic	$-$	$-$	$+$	$+$
nondeterministic	$+$	$-$	$+$	$+$
universal	$-$	$+$	$+$	$+$
alternating	$+$	$+$	$+$	$+$

Acceptance of a word α by an alternating Büchi automaton can also be characterized by a game:

- Positions of Player 0: $V_0 = S \times \omega$;
- Positions of Player 1: $V_1 = 2^S \times \omega$;
- Edges: $\{((s,i),(X,i)) \mid X \models \delta(s,\alpha(i))\}$
  $\cup \{((X,i),(s,i+1)) \mid s \in X\}$

Player 0 wins a play iff $F \times \omega$ is visited infinitely often.

The word α is accepted iff Player 0 has a strategy to win the game from position $(s_0, 0)$.