Automata, Games & Verification

Summary #4

Today at 4:00pm in SR 014

Games in Verification and Synthesis

Walid Haddad: *Model checking games*

(*Simulation Games* postponed to June 12.)

Complementation

Theorem 1. *For each Büchi automaton $\mathcal{A}$ there exists a Büchi automaton $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$.*

Definition 1. *Let $\mathcal{A} = (S, I, T, F)$ be a nondeterministic Büchi automaton. The run DAG of $\mathcal{A}$ on a word $\alpha \in \Sigma^\omega$ is the directed acyclic graph $G = (V, E)$ where*

- $V = \bigcup_{l \geqslant 0} (S_l \times \{l\})$ *where* $S_0 = I$ *and* $S_{l+1} = \bigcup_{s \in S_l, (s, \alpha(l), s') \in T} \{s'\}$
- $E = \{(\langle s, l \rangle, \langle s', l+1 \rangle) \mid l \geqslant 0, (s, \alpha(l), s') \in T\}$

A path in a run DAG is accepting iff it visits F infinitely often.
The automaton accepts α if some path is accepting.

Muller Automata

Definition 2. *A (nondeterministic) Muller automaton* $\mathcal{A}$ *over alphabet* Σ *is a tuple* (S, I, T, F)*:*

- S, I, T *: defined as before*
- $\mathcal{F} \subseteq 2^S$ *: set of accepting subsets, called the table.*

Definition 3. *A run* r *of a Muller automaton is accepting iff* $In(r) \in F$

Theorem 2. *For every (deterministic) Büchi automaton $\mathcal{A}$, there is a (deterministic) Muller automaton $\mathcal{A}'$, such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.*

Theorem 3. *For every nondeterministic Muller automaton $\mathcal{A}$ there is a nondeterministic Büchi automaton $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.*

Theorem 4. *The languages recognizable by deterministic Muller automata are closed under boolean operations.*

Theorem 5. *A language $\mathcal{L}$ is recognizable by a deterministic Muller automaton iff $\mathcal{L}$ is a boolean combination of languages $\overrightarrow{W}$ where $W \subseteq \Sigma^*$ is regular.*

Parity, Rabin, Streett

- A parity automaton is a tuple $(S, I, T, c : S \to \mathbb{N})$.
  A run r of a parity automaton is accepting iff
  $\max\{c(s) \mid s \in In(r)\}$ is even.

- A Rabin automaton is a tuple $(S, I, T, \{(A_i, R_i) \mid i \in J\})$.
  A run r of a Rabin automaton is accepting iff,
  for some $i \in J$, $In(r) \cap A_i \neq \varnothing$ and $In(r) \cap R_i = \varnothing$.

- A Streett automaton is a tuple $(S, I, T, \{(A_i, R_i) \mid i \in J\})$.
  A run r of a Streett automaton is accepting iff,
  for all $i \in J$, $In(r) \cap A_i \neq \varnothing$ or $In(r) \cap R_i = \varnothing$.

Theorem 6. [Tutorial] *Büchi, Muller, Rabin, Streett and parity automata are equally expressive.*

Theorem 7. [Tutorial] *Deterministic Muller, Rabin, Streett and parity automata are equally expressive.*