Limit Your Consumption!
Finding Bounds in Average-energy Games

Joint work with Kim G. Larsen and Simon Laursen (Aalborg University)

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Saarland University

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QAPL 16

Motivation

- Shift from programs to reactive systems:
  - non-terminating
  - interacting with a possibly antagonistic environment
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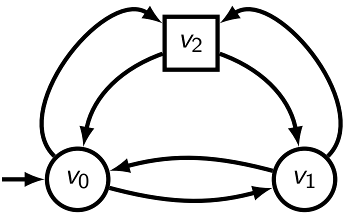
- Here: graph-based games with quantitative winning conditions modeling consumption of a ressource

An Example



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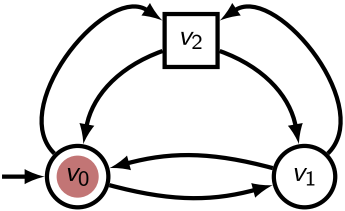
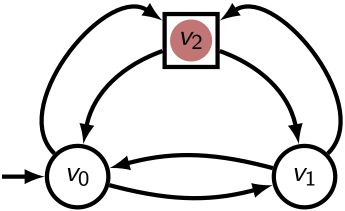


A play:

v_0

An Example



A play:

v_0 v_2

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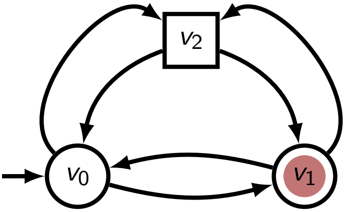

A play:

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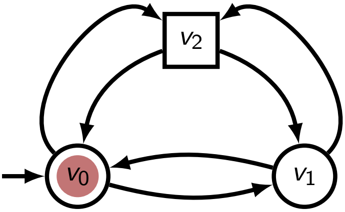

A play:

$v_0 \quad v_2 \quad v_1 \quad v_0$

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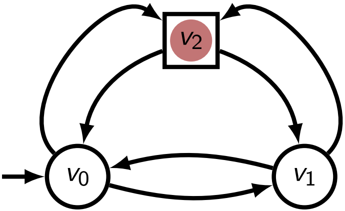

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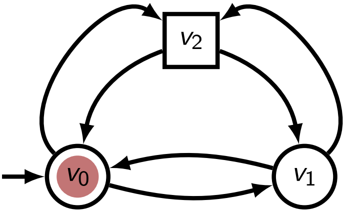

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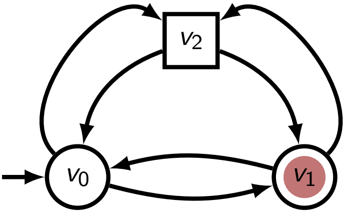

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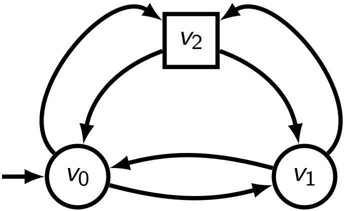

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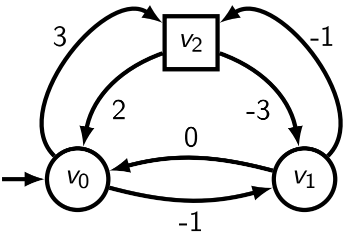

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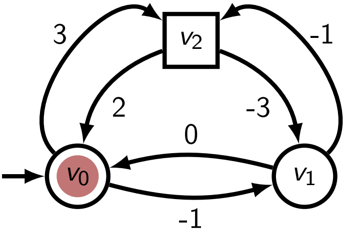

A play (with energy levels):

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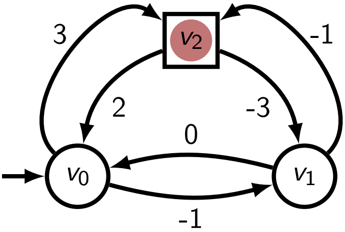

A play (with energy levels):

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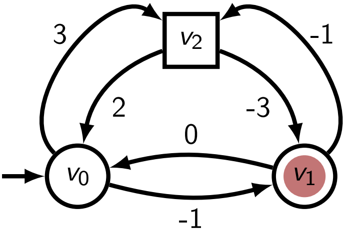

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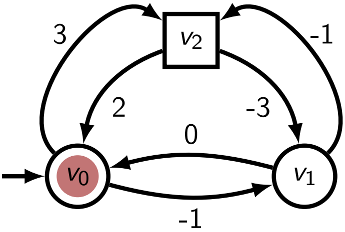

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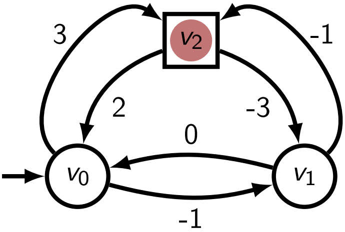

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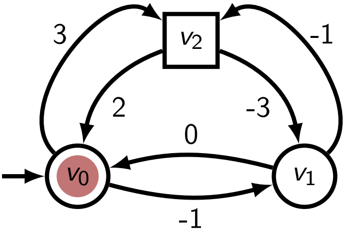

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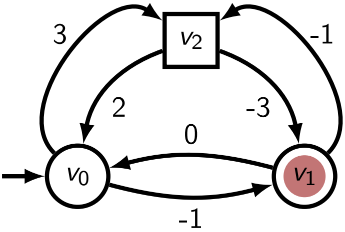

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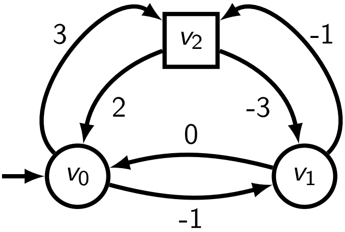

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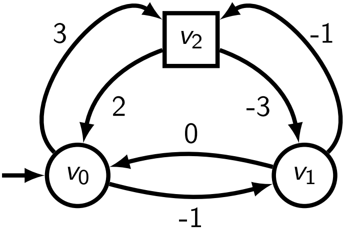

A strategy:

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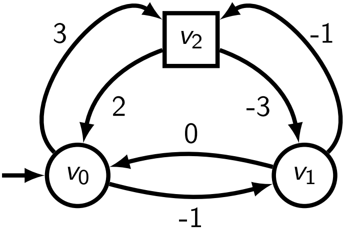

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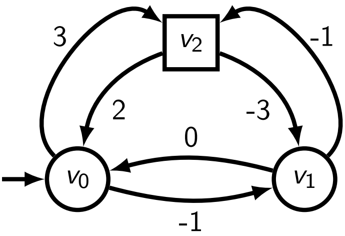

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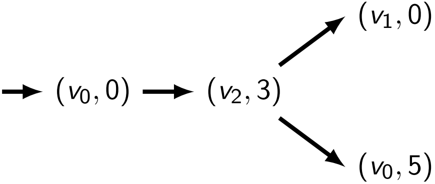

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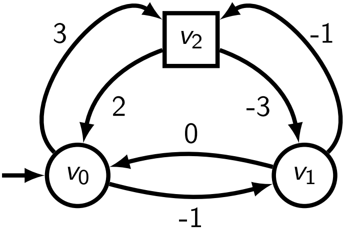

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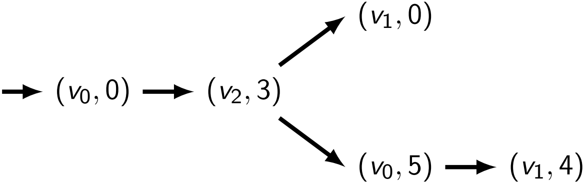

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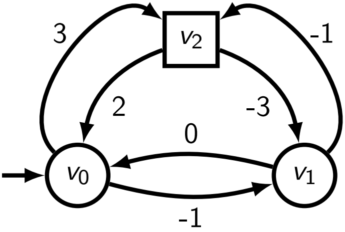

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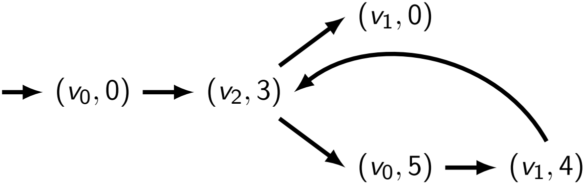

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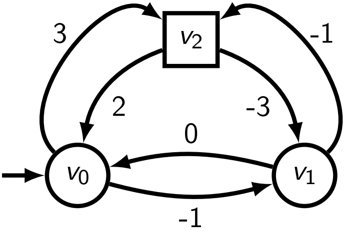

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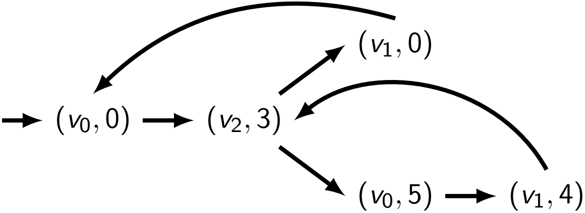

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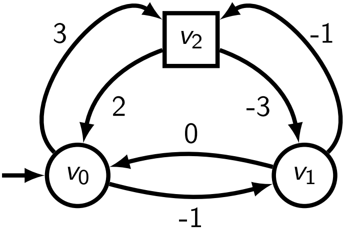

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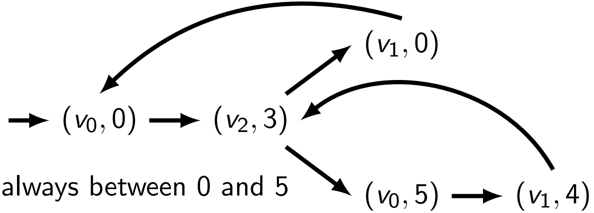

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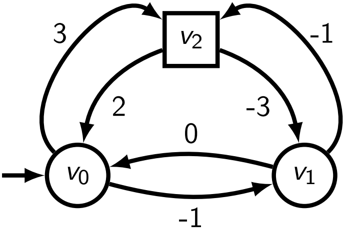

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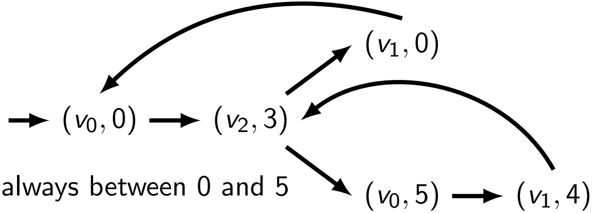

- Energy level always between 0 and 5
- Average energy level at most 4

Previous Work

objective	Complexity	Memory Requirements
EG_L	NP ∩ co-NP	memoryless
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- W.l.o.g.: fix lower bound 0
- In all problems, lower and upper bounds part of the input.
- Here: upper bound existentially quantified.

Objectives

Capacity $cap \in \mathbb{N}$, threshold $t \in \mathbb{N}$

- $\mathsf{EG_L} = \{v_0 v_1 \cdots \mid \forall n.\, 0 \leq \mathrm{EL}(v_0 \cdots v_n)\}$
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Finding Bounds in Average-energy Games

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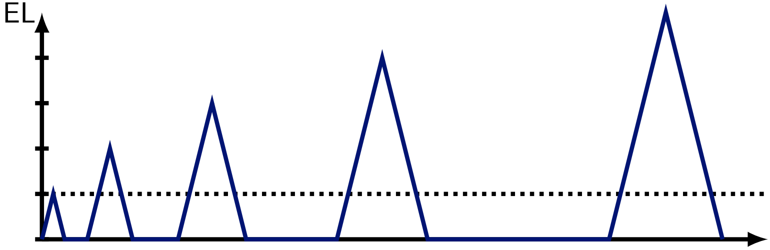
Note:
The direction $\exists cap \Rightarrow \exists t$ is trivial.

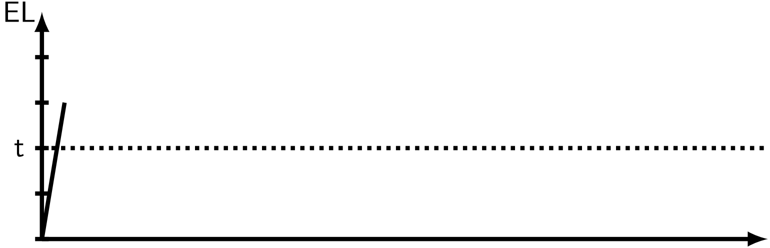
$\exists t \Rightarrow \exists cap$

- Obstacle: average can be bounded while energy level is unbounded



$\exists t \Rightarrow \exists cap$

- But: every time energy level increases above threshold t on average, it drops below t later
- Crossings are characterized by vertex v and energy level in range $t+1, \ldots, t+W$
- For every such combination play like in situation with smallest maximal energy level before next drop below t



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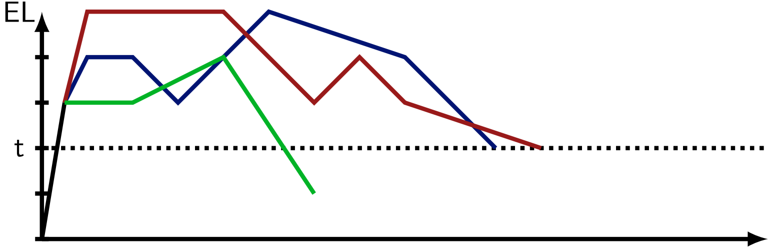
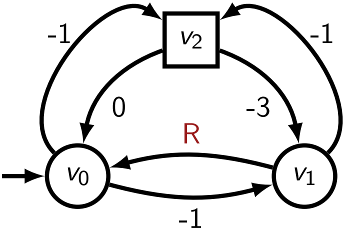


- This strategy bounds the energy level by some *cap*.

Recharge Games

- Previsouly: positive and negative weights
- Now: only negative weights and recharge edges that recharge to a fixed capacity *cap*.



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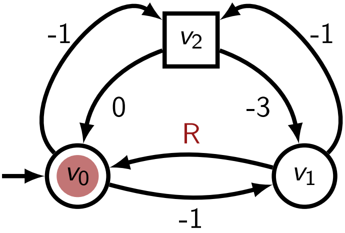

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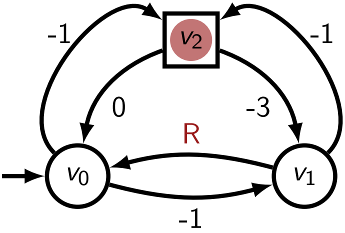

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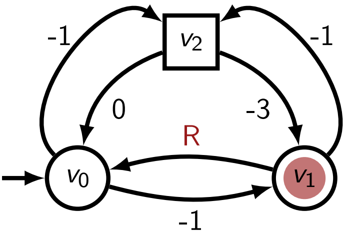

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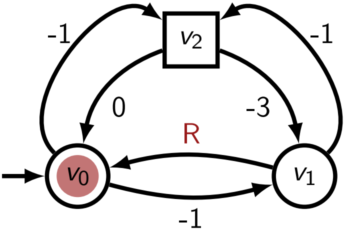

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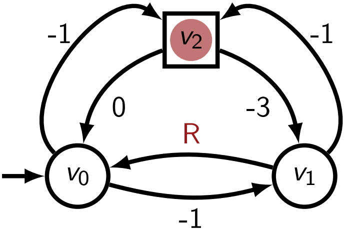

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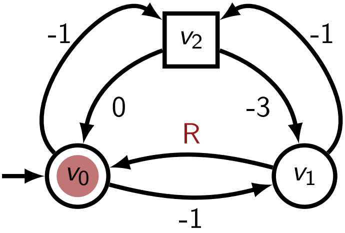

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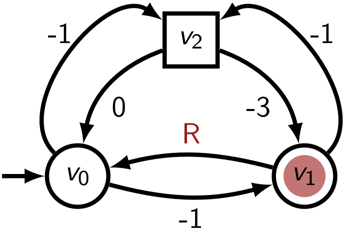

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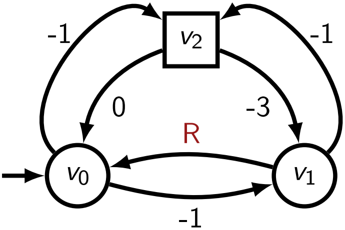

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Objectives

- $\mathsf{RE}(cap) = \{v_0 v_1 \cdots \mid \forall n.\, \mathrm{EL}_{cap}(v_0 \cdots v_n) \geq 0\}$
- $\mathsf{AR}(cap, t) = \mathsf{RE}(cap) \cap$
  $\{v_0 v_1 \cdots \mid \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathrm{EL}_{cap}(v_0 \cdots v_i) \leq t\}$

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Theorem

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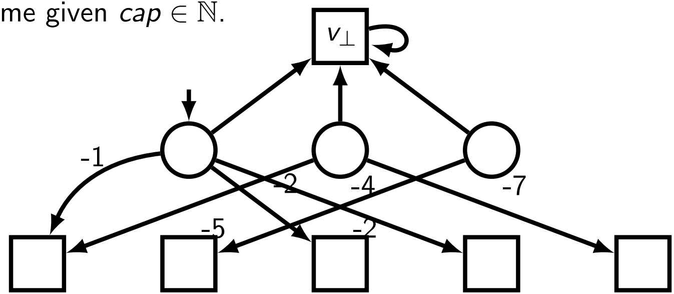
Proof:

Upper bound: Reduction to mean-payoff games.
Lower bound: Reduction from countdown games.

Proof Sketch

A countdown game. Objective: reach $v_\perp$ with energy-level $-cap$ for some given $cap \in \mathbb{N}$.



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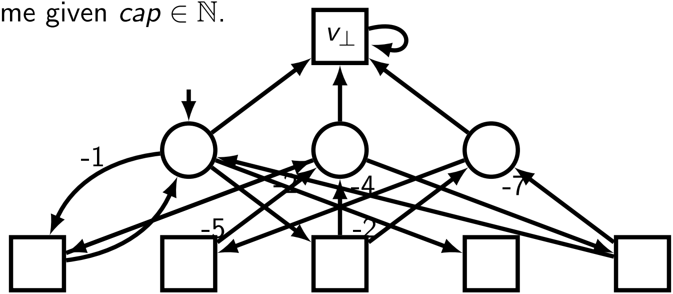

Theorem (Jurdziński, Sproston, Laroussini '08)

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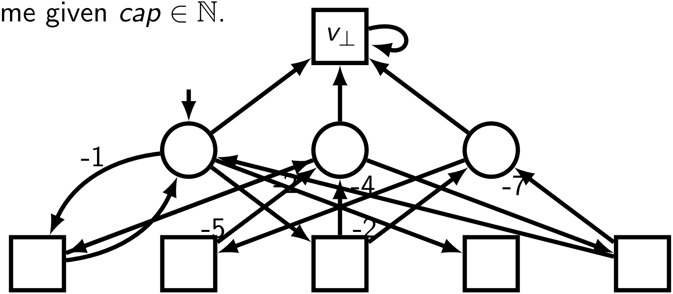

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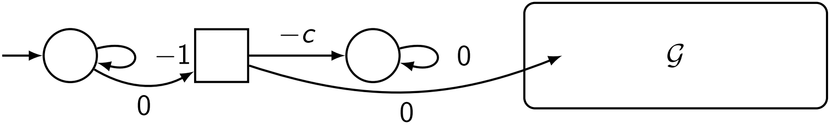
Turn countdown game into average bounded recharge game: capacity *cap* and threshold 0.

Who is to Blame?

Theorem

Solving average-bounded recharge games with existentially quantified capacity and a given threshold is ExpTime*-hard.*



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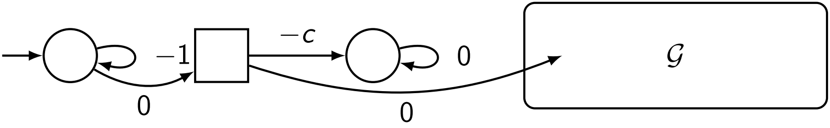


Theorem

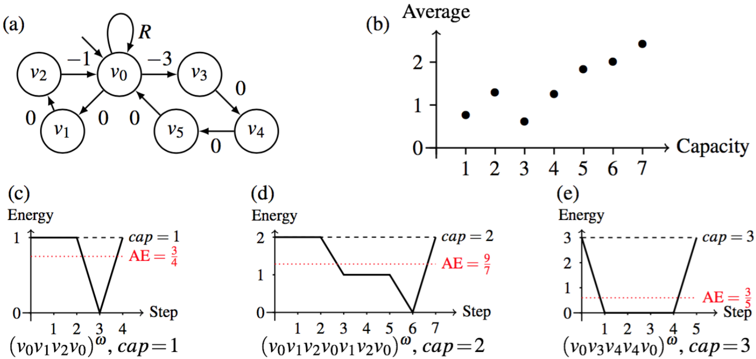
The problem

Input: *Weighted arena* $\mathcal{A}$

Question: *Exists a capacity cap s.t. Player* 0 *wins* $(\mathcal{A}, \mathrm{RE}(\mathit{cap}))$*?*

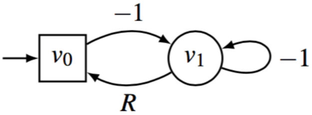
is in $\textsc{PTime}$.

Tradeoffs: Capacity vs. Average

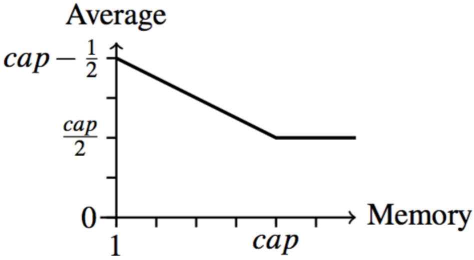


- Available loops depend on capacity
- Tradeoff not monotonic
- Cause of tradeoff: recharge to *cap* at recharge-edges

Tradeoffs: Average vs. Memory



- With n memory states, use self-loop $n-1$ times
- Then, recharge to level cap



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- Multi-dimensional games