Prompt and Parametric LTL Games
- Extended Abstract -

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1 Introduction

Two-player graph games of infinite duration are a tool to synthesize finite-state controllers for reactive systems, i.e., systems which have to interact with an (possibly antagonistic) environment. A concise way to describe requirements on the controlled system (the winning condition of the game) is to use Linear Temporal Logic (LTL). However, LTL is not suitable to express timing constraints, e.g., bounds for eventualities $\mathbf{F}\varphi$. While it is possible to introduce the operator $\mathbf{F}_{\leq k}$ for some fixed bound $k \in \mathbb{N}$ (with the obvious semantics), this does not increase the expressiveness of LTL. Even worse, the bound k is generally not known beforehand and depends on the granularity of the model of the system. Hence, this is not a reasonable way to express timing constraints.

To overcome these shortcomings, extensions of LTL with variable bounds were introduced [1, 2, 4]. We focus here on two such logics and consider games with winning conditions in these logics. As the bounds are free variables, the synthesis problem turns into an optimization problem: what are the best bounds that allow Player 0 to win the game.

Our work extends previous work on LTL games and on time-optimal strategies for other winning conditions [3, 5].

2 PROMPT − LTL Games

Prompt Linear Temporal Logic (PROMPT−LTL) [4] adds the *prompt-eventually* operator $\mathbf{F_P}$ to LTL. The semantics are defined with respect to a fixed bound k: $\mathbf{F_P}\varphi$ holds, if φ holds within the next k steps. In a PROMPT − LTL game with winning condition φ, the bound k is treated as a free variable: a strategy σ for Player 0 is a winning strategy if there exists a k such that every play consistent with σ satisfies φ with bound k.

Theorem 1. *It is decidable, whether Player* 0 *has a winning strategy for a* PROMPT − LTL *game. If she does, then she has a finite-state winning strategy which is effectively computable.*

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3 PLTL Games

Parametric Linear Temporal Logic (PLTL) [1] adds the operators $\mathbf{F}_{\leq x}$ and $\mathbf{G}_{\leq y}$ to LTL, where x and y are free variables. The semantics are then defined with respect to a *variable valuation* α: $\mathbf{F}_{\leq x}\varphi$ holds if φ holds within the next $\alpha(x)$ steps, and $\mathbf{G}_{\leq y}\varphi$ holds if φ holds at least for the next $\alpha(y)$ steps. Hence, PLTL is in two aspects more expressive than PROMPT−LTL: in PROMPT−LTL, there is a single bound k for all prompt eventualities in φ, while PLTL allows to use different variables. Also, the bounded $\mathbf{G}$ is not expressible in PROMPT − LTL.

PLTL games are defined with respect to a variable valuation α. A strategy σ for Player 0 is a winning strategy with respect to α, if every play consistent with σ satisfies the winning condition φ with respect to α. A strategy τ for Player 1 is a winning strategy with respect to α, if every play consistent with τ does not satisfy φ with respect to α. Player i wins a PLTL game $\mathcal{G}$ with respect to α, if she has a winning strategy. The set $\mathcal{W}^i_{\mathcal{G}}$ contains all such α.

Theorem 2. *Let $\mathcal{G}$ be a* PLTL *game, α a valuation, and $i \in \{0, 1\}$. It is decidable whether*

(i) $\alpha \in \mathcal{W}^i_{\mathcal{G}}$, i.e., whether Player i wins $\mathcal{G}$ with respect to α.
(ii) $\mathcal{W}^i_{\mathcal{G}}$ is non-empty, infinite, or universal.

The fragments $\mathrm{PLTL}_{\mathbf{F}}$ and $\mathrm{PLTL}_{\mathbf{G}}$ are defined syntactically by adding only $\mathbf{F}_{\leq x}$ respectively only $\mathbf{G}_{\leq y}$ to LTL. For these fragments, it makes sense to view synthesis of winning strategies as an optimization problem: what is the *best* variable valuation α such that Player 0 wins $\mathcal{G}$ with respect to α.

Theorem 3. *Let $\mathcal{G}_{\mathbf{F}}$ be a $\mathrm{PLTL}_{\mathbf{F}}$ game and $\mathcal{G}_{\mathbf{G}}$ be a $\mathrm{PLTL}_{\mathbf{G}}$ game. Then, the following optimization problems can be solved effectively.*

(i) Determine $\min_{\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{F}}}} \min_{x \in \mathrm{var}(\varphi)} \alpha(x)$.
(ii) Determine $\min_{\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{F}}}} \max_{x \in \mathrm{var}(\varphi)} \alpha(x)$.
(iii) Determine $\max_{\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{G}}}} \max_{y \in \mathrm{var}(\varphi)} \alpha(y)$.
(iv) Determine $\max_{\alpha \in \mathcal{W}^0_{\mathcal{G}_{\mathbf{G}}}} \min_{y \in \mathrm{var}(\varphi)} \alpha(y)$.

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