Degrees of Lookahead in Context-free Infinite Games*

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Motivation. We continue the investigation of delay games, infinite games in which one player may postpone her moves for some time to obtain a lookahead on her opponents moves. Hosch and Landweber proved that a delay game with regular winning condition is won by the player with lookahead if and only if she can win with (triply-exponential) constant delay, i.e., the lookahead which is necessary to win is bounded [1].

Walukiewicz showed that games (without delay) with deterministic context-free winning conditions can be solved effectively [2]. We investigate the algorithmic properties of delay games with deterministic context-free winning conditions as well as lower bounds on the necessary delay.

Results. Given a function $f\colon \mathbb{N} \to \mathbb{N}_+$ and an ω-language $L \subseteq (\Sigma_I \times \Sigma_O)^\omega$, the game $\Gamma_f(L)$ is played by two players (the input player I and the output player O) in rounds $i = 0, 1, 2, \ldots$ as follows: in round i, Player I picks a word $u_i \in \Sigma_I^{f(i)}$, then Player O picks one letter $v_i \in \Sigma_O$. Define $\alpha = u_0u_1u_2\cdots$ and $\beta = v_0v_1v_2\cdots$. Player O wins if $\binom{\alpha(0)}{\beta(0)}\binom{\alpha(1)}{\beta(1)}\binom{\alpha(2)}{\beta(2)}\cdots \in L$.

A function $f\colon \mathbb{N} \to \mathbb{N}_+$ is a *constant delay function*, if $f(i) = 1$ for all $i > 0$; f is a *linear delay function*, if $f(i) = k > 0$ for all $i \geq 0$; and f is an *elementary delay function*, if $\sum_{i=0}^{n} f(i) \in \mathcal{O}(\exp_k)$, where $\exp_k$ is the k-fold exponential with base 2. Given an ω-language L, we say that Player p *wins the game induced by L with constant, linear, elementary,* or *finite delay*, if there exists a constant, linear, elementary, or arbitrary delay function f such that Player p has a winning strategy for $\Gamma_f(L)$.

Theorem 1. *The following problem is undecidable:*
Input: *A deterministic ω-PDA $\mathcal{A}$ with parity acceptance condition.*
Question: *Does Player O win the game induced by $L(\mathcal{A})$ with finite delay?*

This result even holds when we replace "finite delay" by "constant delay" or "linear delay" and for restricted classes of automata. Concerning lower bounds on the delay necessary to win we obtain the following.

Theorem 2. *There exists a deterministic ω-context-free language L such that Player O wins the game induced by L with finite delay, but for any elementary delay function f, the game $\Gamma_f(L)$ is won by Player I.*

References

1. Hosch, F., Landweber, L.H.: Finite delay solutions for sequential conditions. In: Nivat, M. (ed.), ICALP 1972, pp. 45–60. North-Holland, Amsterdam (1972)
2. Walukiewicz, I.: Pushdown processes: games and model-checking. Inf. Comput., 164(2), 234–263 (2001)

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