From LTL to rLTL Monitoring

Maximilian Schwenger
Joint Work With Corto Mascle, Daniel Neider, Paulo Tabuada, Alexander Weinert, Martin Zimmermann
Why rLTL rather than LTL?

Assumption $\implies$ Guarantee

View Always Unobstructed $\implies$ Always Stay on Lane

$G(\text{unobs. view}) \implies G(\text{on lane})$
Why rLTL rather than LTL?

Assumption $\iff$ Guarantee

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$G(\text{unobs. view}) \iff G(\text{on lane})$

Problem 1: One Frame Camera Glitch $\iff$ Do Whatever You Want
**Why rLTL rather than LTL?**

Assumption $\implies$ Guarantee

View Always Unobstructed $\implies$ Always Stay on Lane

$G(\text{unobs. view}) \implies G(\text{on lane})$

Problem 1: One Frame $\implies$ Do Whatever

Camera Glitch $\implies$ You Want

Problem 2: Crash Immediately $\iff$ Drive Perfectly
**Why rLTL rather than LTL?**

LTL to rLTL:
- More Robustness
- More Information

- View Always Unobstructed $\iff$ Always Stay on Lane
- Always Stay on Lane $\iff$ Guarantee
- Guarantee $\iff$ Assumption

**Problem 1:**
- One Frame
  - Camera Glitch
- Do Whatever You Want

**Problem 2:**
- Crash Immediately
- Drive Perfectly

Guarantee Assumption $\iff$ Always Stay on Lane

G(unobs. view) $\iff$ G(on lane)
Lift Monitoring from LTL to rLTL

- rLTL on Finite Traces
- Construction of an rLTL Monitor
- Case Study: LTL v. rLTL
Lift Monitoring from LTL to rLTL

1. rLTL on Finite Traces
2. Construction of an rLTL Monitor
3. Case Study: LTL v. rLTL
What you need to know about (r)LTL semantics

\[ a \in \Sigma, \quad \text{AP} = 2^\Sigma, \quad \text{trace } \pi \in \text{AP}^\omega \]

**Example**

\[ \varphi \equiv a \quad \pi = \{a\} * * * \]

What you need to know about (r)LTL semantics

\[ a \in \Sigma, \quad \text{AP} = 2^\Sigma, \quad \text{trace } \pi \in \text{AP}^\omega \]

Example

\[ \varphi \equiv a \]

\[ \varphi \equiv G a \]

\[ \pi = \{a\} * * * * \]

\[ \pi = \{a\} \{a\} \{a\} \{a\} \]
LTL

\( a \in \Sigma \), \( \text{AP} = 2^\Sigma \), \( \text{trace } \pi \in \text{AP}^\omega \)

Example

\[ \varphi \equiv a \]
\[ \pi = \{a\} * * * * \]

\[ \varphi \equiv G a \]
\[ \pi = \{a\}\{a\}\{a\}\{a\} \]

\[ \varphi \equiv F a \]
\[ \pi = \{\} \{\} \{a\} * \]

What you need to know about (r)LTL semantics

\[ a \in \Sigma, \quad AP = 2^\Sigma, \quad \text{trace } \pi \in AP^\omega \]

Example

\[ \varphi \equiv a \]
\[ \pi = \{a\} * * * \]

\[ \varphi \equiv G a \]
\[ \pi = \{a\}\{a\}\{a\}\{a\} \]

\[ \varphi \equiv F a \]
\[ \pi = \{\} \{\} \{a\} * \]

Output: 1/0

What you need to know about (r)LTL semantics

\[ \phi \equiv G \alpha, \quad \pi = \{a\} \{a\} \{a\} \{a\} \]

Example

Tabuada, Neider. "Robust linear temporal logic". CSL 2016
What you need to know about (r)LTL semantics

\[ a \in \Sigma, \quad \text{AP} = 2^\Sigma, \quad \text{trace } \pi \in \text{AP}^\omega \]

Example

\[ \varphi \equiv G a \]

\[ \pi = \{ a \} \{ a \} \{ a \} \{ a \} \]

"Ga" "FGa" "GFa" "Fa"
What you need to know about (r)LTL semantics

\[ a \in \Sigma, \quad AP = 2^\Sigma, \quad \text{trace } \pi \in AP^\omega \]

Example

\[ \varphi \equiv Ga \]

\[ \pi = \{a\} \{a\} \{a\} \{a\} \]

Output:

1/0 1/0 1/0 1/0
Finite Semantics: Ternary Output

1 – Already Satisfied  0 – Already Falsified  ? – Don’t Know
Finite Semantics: Ternary Output

<table>
<thead>
<tr>
<th>Formula</th>
<th>Prefix</th>
<th>LTL</th>
<th>rLTL ((G, FG, GF, F))</th>
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<tbody>
<tr>
<td>(G{a})</td>
<td>(\varepsilon)</td>
<td>(?)</td>
<td>(????)</td>
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<tr>
<td>(G{a})</td>
<td>{a}</td>
<td>(?)</td>
<td>(???1)</td>
</tr>
<tr>
<td>(G{a})</td>
<td>{a}}</td>
<td>(0)</td>
<td>(0??1)</td>
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1 – Already Satisfied  
0 – Already Falsified  
? – Don’t Know

### Finite Semantics: Ternary Output

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<td>?</td>
<td>????</td>
</tr>
<tr>
<td>({a})</td>
<td>({a})</td>
<td>?</td>
<td>????1</td>
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<tr>
<td>({a}{})</td>
<td>0</td>
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<td>0???1</td>
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</table>

Questions: What truth values might occur?

Finite Semantics: Realizable Verdicts

<table>
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<tr>
<th>Value</th>
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<tbody>
<tr>
<td>0000</td>
<td>$\varepsilon$</td>
<td>$a \land \neg a$</td>
<td>0?11</td>
<td>$\emptyset{a}$</td>
<td>$\Box a \lor \Box \neg a$</td>
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<tr>
<td>000?</td>
<td>$\varepsilon$</td>
<td>$\Diamond \Box a \land \Diamond \neg \Diamond a$</td>
<td>0111</td>
<td>$\emptyset{a}$</td>
<td>$a \mathcal{R} a$</td>
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<tr>
<td>0001</td>
<td>unrealizable</td>
<td></td>
<td>0???1</td>
<td>${a}$</td>
<td>$\Box a$</td>
</tr>
<tr>
<td>00??</td>
<td>$\varepsilon$</td>
<td>$\Box a \land \Box \neg a$</td>
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<td>$\emptyset{a}$</td>
<td>$\Box a \land \Diamond \neg \Diamond a$</td>
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<tr>
<td>0011</td>
<td>unrealizable</td>
<td></td>
<td>??11</td>
<td>$\varepsilon$</td>
<td>$\Box a \lor \Diamond \neg \Diamond a$</td>
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<tr>
<td>0??1</td>
<td>$\emptyset{a}$</td>
<td>$\Box a$</td>
<td>??11</td>
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<td>$\Box a \lor \Diamond \neg \Diamond \neg a$</td>
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<tr>
<td>0??2</td>
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<td>$\Box a$</td>
<td>?111</td>
<td>$\varepsilon$</td>
<td>$a \lor \neg a$</td>
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<tr>
<td>0??3</td>
<td>$\emptyset{a}$</td>
<td>$\Box a$</td>
<td>1111</td>
<td>$\varepsilon$</td>
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## Finite Semantics: Realizable Verdicts

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<td>0000</td>
<td>ε</td>
<td>(a \land \neg a)</td>
<td>0?11</td>
<td>(\emptyset{a})</td>
<td>(\square a \lor \lnot a)</td>
</tr>
<tr>
<td>000?</td>
<td>ε</td>
<td>(\lozenge \square a \land \lozenge \neg \lozenge a)</td>
<td>0111</td>
<td>(\emptyset{a})</td>
<td>(a \mathcal{R} a)</td>
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<td>0001</td>
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<tr>
<td>00??</td>
<td>ε</td>
<td>(\square a \land \square \neg a)</td>
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**Theorem:** An rLTL Monitor cannot yield 0001 nor 0011.
### Finite Semantics: Ternary Output

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<tr>
<td>Ga</td>
<td>ε</td>
<td>?</td>
<td>?????</td>
</tr>
<tr>
<td></td>
<td>{a}</td>
<td>?</td>
<td>???1</td>
</tr>
<tr>
<td></td>
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1 – Already Satisfied | 0 – Already Falsified | ? – Undetermined

**Questions:** How do values “evolve”?
**Finite Semantics: Ternary Output**

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**Questions:** How do values “evolve”?  

**Theorem:** Up to four refinements are possible.
**Monitorability**

**rLTL-Ugly Prefix**: Every continuation yields $???

**rLTL-Monitorable**: There are no **rLTL-Ugly Prefixes**

<table>
<thead>
<tr>
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<th>LTL Monitorable</th>
<th>Not LTL Monitorable</th>
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<tr>
<td><strong>rLTL Monitorable</strong></td>
<td>$G_a$</td>
<td>$GF_a$</td>
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<tr>
<td>Not <strong>rLTL Monitorable</strong></td>
<td>$(G_a \land \neg a) \implies (FG_a \land \neg F\neg a)$</td>
<td>$(p \land \varphi_{LTL}) \lor (\neg p \land \varphi_{rLTL})$</td>
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Monitorability

rLTL-Ugly Prefix: Every continuation yields ???

rLTL-Monitorable: There are no rLTL-Ugly Prefixes

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| rLTL Monitorable | rLTL: “Adding {} will always yield 0***”  
LTL: “Adding {} will always yield 0”  
Ga  | GFa |
| Not rLTL Monitorable | (Ga ∧ G¬a) ⟹ (FGa ∧ FG¬a)  | (p ∧ φLTL) ∨ (¬p ∧ φrLTL) |
**Monitorability**

**rLTL-Ugly Prefix:** Every continuation yields ***

**rLTL-Monitorable:** There are no rLTL-Ugly Prefixes

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<td></td>
<td>$\text{Ga}$</td>
<td>$\text{GFa}$</td>
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<tr>
<td>LTL: “Adding {} will always yield 0”</td>
<td>LTL: “Depends on infinite behavior.”</td>
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<td><strong>Not rLTL Monitorable</strong></td>
<td>$(\text{Ga} \land \neg \text{a}) \implies (\text{FGa} \land \text{FG¬a})$</td>
<td>$(p \land \varphi_{\text{LTL}}) \lor (\neg p \land \varphi_{\text{rLTL}})$</td>
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LTL: “Adding {} will always yield 0”

rLTL: “Adding {a} will yield 1 in last bit”
LTL-Mon does not imply rLTL-Mon

rLTL-Ugly Prefix: Every continuation yields ???

rLTL-Monitorable: There are no rLTL-Ugly Prefixes

\[(Ga \land G\neg a) \iff (FGa \land FG\neg a)\]

LTL-mon

(Ga \land G\neg a): Contradiction

(Ga \land G\neg a) \implies (FGa \land FG\neg a): Tautology

Not rLTL-mon

Ugly Prefix \{ \{a\}\}

\forall \rho: \{ \{a\}\rho\{ \}^\omega \text{ yields } 1111 \}

\{ \{a\}\rho\{a\}^\omega \text{ yields } 0000 \}
### Monitorability

**rLTL-Ugly Prefix:** Every continuation yields ????

**rLTL-Monitorable:** There are no rLTL-Ugly Prefixes

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<tr>
<td>Not rLTL Monitorable</td>
<td>(p∧φ_{LTL}) ∨ (¬p ∧ φ_{rLTL})</td>
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- LTL: “Adding {} will always yield 0”  
- LTL: “Adding {} will always yield 0***”  
- rLTL: “Adding {} will yield 1 in last bit”  
- (Ga ∧ G¬a) → (FGa ∧ FG¬a)
Lift Monitoring from LTL to rLTL

- rLTL on Finite Traces
- Construction of an rLTL Monitor
- Case Study: LTL v. rLTL
Lift Monitoring from LTL to rLTL

rLTL on Finite Traces

Construction of an rLTL Monitor

Case Study: LTL v. rLTL
Constructing an rLTL Monitor

rLTL

\( \varphi \)
Constructing an rLTL Monitor

rLTL

Büchi

\( \mathcal{A}_G \)

\( \mathcal{A}_{FG} \)

\( \mathcal{A}_{GF} \)

\( \mathcal{A}_F \)

\( \mathcal{A}_{G\neg} \)
Constructing an rLTL Monitor

\( rLTL \) \hspace{1cm} \text{B"uchi}

\[ \begin{align*}
\mathcal{A}_G \\
\mathcal{A}_{FG} \\
\mathcal{A}_{GF} \\
\mathcal{A}_F \\
\mathcal{A}_{G\neg} \\
\end{align*} \]

\( |\varphi| \)
Constructing an rLTL Monitor

Ga

FGa

GFa

Fa

G¬a
Constructing an rLTL Monitor

\[ \mathcal{A}_{\neg} \]

\[ \mathcal{A}_G \]

\[ \mathcal{A}_{FG} \]

\[ \mathcal{A}_{GF} \]

\[ \mathcal{A}_F \]

\[ 2^\mathcal{O}(|\varphi|) \]

rLTL

Büchi
Constructing an rLTL Monitor

\[ rLTL \xrightarrow{\varphi} \mathcal{A} \xrightarrow{G} \mathcal{B} \xrightarrow{F} \mathcal{G} \]

\[ 2^O(|\varphi|) \]
Constructing an rLTL Monitor

\[ \phi \]

- \[ A_G \rightarrow B_G \]
- \[ A_{FG} \rightarrow B_{FG} \]
- \[ A_{GF} \rightarrow B_{GF} \]
- \[ A_F \rightarrow B_F \]
- \[ A_{G\neg} \rightarrow B_{G\neg} \]

\[ 2^{O(|\phi|)} \quad O(|A|^3) \]
Constructing an rLTL Monitor

<table>
<thead>
<tr>
<th>rLTL</th>
<th>Büchi</th>
<th>NFA</th>
<th>Det. Moore</th>
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<tbody>
<tr>
<td>( \varphi )</td>
<td>( \mathcal{A} )</td>
<td>( \mathcal{B} )</td>
<td>( \mathcal{C} )</td>
</tr>
<tr>
<td>( \mathcal{A}_G )</td>
<td>( \mathcal{B}_G )</td>
<td>( \mathcal{C}_G )</td>
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<tr>
<td>( \mathcal{A}_{FG} )</td>
<td>( \mathcal{B}_{FG} )</td>
<td>( \mathcal{C}_{FG} )</td>
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\[ 2^{O(|\varphi|)} \quad O(|\mathcal{A}|^3) \]
Constructing an rLTL Monitor

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<td>$\mathcal{C}_G$</td>
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<td>$\mathcal{B}_{FG}$</td>
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<td>$\mathcal{C}_{G\neg}$</td>
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$2^O(|\varphi|)$  $O(|\mathcal{A}|^3)$  $2^O(|\mathcal{B}|)$
Constructing an rLTL Monitor

Ga, FGa, GFa, Fa, G¬a

Büchi DFA
Constructing an rLTL Monitor

rLTL | Büchi | NFA | Det. Moore
--- | --- | --- | ---
\( \mathcal{A} \) | \( \mathcal{B} \) | \( \mathcal{C} \) | \( \mathcal{D} \)
\( \mathcal{A} \mathcal{G} \) | \( \mathcal{B} \mathcal{G} \) | \( \mathcal{C} \mathcal{G} \) | \( \mathcal{D} \mathcal{G} \)
\( \mathcal{A} \mathcal{F} \mathcal{G} \) | \( \mathcal{B} \mathcal{F} \mathcal{G} \) | \( \mathcal{C} \mathcal{F} \mathcal{G} \) | \( \mathcal{D} \mathcal{F} \mathcal{G} \)
\( \mathcal{A} \mathcal{G} \mathcal{F} \) | \( \mathcal{B} \mathcal{G} \mathcal{F} \) | \( \mathcal{C} \mathcal{G} \mathcal{F} \) | \( \mathcal{D} \mathcal{G} \mathcal{F} \)
\( \mathcal{A} \mathcal{F} \) | \( \mathcal{B} \mathcal{F} \) | \( \mathcal{C} \mathcal{F} \) | \( \mathcal{D} \mathcal{F} \)
\( \mathcal{A} \mathcal{G} \neg \) | \( \mathcal{B} \mathcal{G} \neg \) | \( \mathcal{C} \mathcal{G} \neg \) | \( \mathcal{D} \mathcal{G} \neg \)

\( 2^{O(|\varphi|)} \) | \( O(|\mathcal{A}|^3) \) | \( 2^{O(|\mathcal{B}|)} \)
Constructing an rLTL Monitor

- rLTL
- Büchi
- NFA
- Det. Moore
- Det. Moore

\[
\phi \xrightarrow{A_G} B_G \xrightarrow{C_G} \\
\phi \xrightarrow{A_{FG}} B_{FG} \xrightarrow{C_{FG}} \\
\phi \xrightarrow{A_{GF}} B_{GF} \xrightarrow{C_{GF}} \\
\phi \xrightarrow{A_F} B_F \xrightarrow{C_F} \\
\phi \xrightarrow{A_{G\neg}} B_{G\neg} \xrightarrow{C_{G\neg}} \\
\]

\[
2^{O(|\phi|)} \quad O(|A|^3) \quad 2^{O(|B|)}
\]
Constructing an rLTL Monitor

\[ \varphi \]

\[ A_G \rightarrow B_G \rightarrow C_G \]
\[ A_{FG} \rightarrow B_{FG} \rightarrow C_{FG} \]
\[ A_{GF} \rightarrow B_{GF} \rightarrow C_{GF} \]
\[ A_F \rightarrow B_F \rightarrow C_F \]
\[ A_{G\neg} \rightarrow B_{G\neg} \rightarrow C_{G\neg} \]

\[ 2^{O(|\varphi|)} \]
\[ O(|A|^3) \]
\[ 2^{O(|B|)} \]
\[ O(|C|) \]
Constructing an rLTL Monitor

Ga

FGa

GFa

Fa

G¬a

Result

1111
0111
0011
0001
0000
????

1111
0111
0011
0001
0000
????

1111
0111
0011
0001
0000
???1

1111
0111
0011
0001
0000
0??1
Constructing an rLTL Monitor

\[ \phi \]

\( \mathcal{A} \)

\( \mathcal{B} \)

\( \mathcal{C} \)

\( \mathcal{M}_\phi \)

\( 2^O(|\phi|) \)

\( O(|A|^3) \)

\( 2^O(|B|) \)

\( O(|C|) \)
Constructing an rLTL Monitor

\[ \begin{align*}
2^O(|\varphi|) & \quad O(|\mathcal{A}|^3) & \quad 2^O(|\mathcal{B}|) & \quad O(|\mathcal{C}|)
\end{align*} \]
Constructing an rLTL Monitor

\[
\varphi \xrightarrow{\mathcal{A}_G} \mathcal{B}_G \xrightarrow{} \mathcal{C}_G \\
\varphi \xrightarrow{\mathcal{A}_{FG}} \mathcal{B}_{FG} \xrightarrow{} \mathcal{C}_{FG} \\
\varphi \xrightarrow{\mathcal{A}_{GF}} \mathcal{B}_{GF} \xrightarrow{} \mathcal{C}_{GF} \\
\varphi \xrightarrow{\mathcal{A}_F} \mathcal{B}_F \xrightarrow{} \mathcal{C}_F \\
\varphi \xrightarrow{\mathcal{A}_{G\neg}} \mathcal{B}_{G\neg} \xrightarrow{} \mathcal{C}_{G\neg}
\]

\[2^{O(|\varphi|)} \quad O(|\mathcal{A}|^3) \quad 2^{O(|\mathcal{B}|)} \quad O(|\mathcal{C}|) \quad O(|\mathcal{M}| \log(|\mathcal{M}|))\]
Constructing an rLTL Monitor


\[
\phi \xrightarrow{A_G} B_G \xrightarrow{C_G} \quad \phi \xrightarrow{A_{FG}} B_{FG} \xrightarrow{C_{FG}} \quad \phi \xrightarrow{A_{GF}} B_{GF} \xrightarrow{C_{GF}} \quad \phi \xrightarrow{A_F} B_F \xrightarrow{C_F} \quad \phi \xrightarrow{A_{G\neg}} B_{G\neg} \xrightarrow{C_{G\neg}}
\]

\[
M_\phi \xrightarrow{M'_\phi}
\]

Total: \(2^{2^{O(|\phi|)}}\)

\(2^{O(|\phi|)}\) \quad \(O(|A|^3)\) \quad \(2^{O(|B|)}\) \quad \(O(|C|)\) \quad \(O(|M|\log(|M|))\)
Lift Monitoring from LTL to rLTL

rLTL on Finite Traces → Construction of an rLTL Monitor → Case Study: LTL v. rLTL
Lift Monitoring from LTL to rLTL

- rLTL on Finite Traces
- Construction of an rLTL Monitor
- Case Study: LTL v. rLTL
Dwyer et al [1]:

97 LTL formulas
frequent specification patterns

Benchmark

Dwyer et al [1]:
97 LTL formulas
frequent specification patterns

55.7% LTL-monitorable [2]  versus  100% rLTL-monitorable

We adapted the three-valued
is undetermined. While an in
speci
nite repetition of the last position of the pre
an undetermined result, the monitor could indicate that an in
execution would satisfy the speci
a state in which
Even though a pre
ther when attributing a special role to the last position(s) of a pre
the middle bits can be relevant. In further work, we will investigate
the other hand, the truth value
provide enough information to distinguish these two formulas. On
represents
0001
its LTL counterpart for 77% of their formulas.

The approach on the benchmark of Bauer et al.: All formulas are
ically no more expensive than the one for
et al. to
3 CONCLUSION
construction time was negligible for almost all benchmarks
although they require considerably more time to construct, the overall
is only constructed once before it is deployed.

Moreover, desired properties for cyber-physical systems often
despite their strong semantics and satis
position of the trace. A formula that is undetermined under the
operator whose operand is consider violated (satis
RV-LTL. Here, the speci
an extension of rLTL which takes this observation into account.

In particular, we designed a new tool for producing monitors for
rLTL which fully satisfies the specification. However, this tool found no
monitor is strictly more informative than
rLTL-mon
monitors. Thus, a real-time extension for
rLTL
that

Moreover, the informedness of a monitor can be increased fur-
3 RESULTS
Finally, our analysis answers our second question:

(a) Analysis of the monitor construction for the
(b) Analysis of the number of monitors with respect to their size

Number of monitors
(97 in total)

Number of states

<table>
<thead>
<tr>
<th>Number of states</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of monitors</td>
<td>43</td>
<td>20</td>
<td>21</td>
<td>10</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

Histogram of the number of monitors with respect to their size

Analysis of the monitor construction for the 54 formulas that are both LTL-monitorable and rLTL-monitorable
We adapted the three-valued logic to monitor properties that are undetermined. While an infinite repetition of the last position of the prefix of a trace might not result in a violation, the monitor could indicate that an undetermined result, the monitoring could reset the monitor. Moreover, desired properties for cyber-physical systems often include real-time components such as "touch the ground at most 10 seconds after receiving a landing command". Monitors for logics that do not fully satisfy strong semantics and satisfiability, such as STL, have a low runtime cost and thus increase its level of informedness.

3 CONCLUSION

In total, the truth value of a preformula can be relevant. In further work, we will investigate the role of the middle bits in rLTL monitoring. The informedness of a monitor can be increased further.

Finally, our analysis answers our second question: Although they require considerably more time to construct, the overall construction time was negligible for almost all benchmarks. Moreover, the informedness of a monitor can be increased further.

The authors would like to thank Li Bingchen for discovering the formula mentioned in Footnote 52. This points to a drawback regarding the approach on the benchmark of Bauer et al. All formulas are monitorable and the computational overhead at runtime when compared to LTL tools is only constructed once before it is deployed. Additionally, our analysis answers our second question: 55.7% LTL-monitorable versus 100% rLTL-monitorable.

The work of Maximilian Schwenke was supported by the Saarbrücken Graduate School of Computer Science. The work of Martin Zimmermann as part of the Collaborative Research Center "Center for Perspicuous Interaction and User Experience" (TRR 248, 389792660). The work of Paulo Tabuada was partially supported by the NSF project 1645824. The work of Alexander Weinert was supported by the OSARES (No. 683300) and the German Research Foundation (DFG).
We adapted the three-valued is undetermined. While an infinite repetition of the last position of the pre-execution would satisfy the specification, even though a pre-execution was supported by the Engineering and Physical Sciences Research Council (EPSRC) EP/S032207/1. The work of Martin Zimmermann was partially supported by the NSF project 1645824. The work of Paulo Tabuada was supported by the Saarbrücken Graduate School of Computer Science. The work of Alexander Weinert was supported by the Collaborative Research Center "Center for Perspicuous Monitoring and Diagnosing Cyber-Physical Systems" (TRR 248, 389792660). The work of Marco Cattari was supported by the OSARES (No. 683300) and the German Research Foundation (DFG) under Grant No. 272813044. The work of1 the authors would like to thank Li Bingchen for discovering the formula mentioned in Footnote 3. It is considered potentially true (potentially false) if the truth value of a position of the trace. A formula that is undetermined under the operator whose operand is considered violated (satisfied) can contain the strong (weak) next-position of the pre-execution.

Moreover, desired properties for cyber-physical systems often include real-time components such as "touch the ground at most 15 seconds after receiving a landing command". Monitors for logics are only constructed once before it is deployed. Their strong semantics and satisfaction are considered rLTL-monitors. Thus, a real-time extension for rLTL which takes this observation into account.

Finally, our analysis answers our second question: how LTL monitors for logics are only slightly larger than the corresponding rLTL-monitors. Note that this is not a concern in practice as a monitor is strictly more informative than its LTL counterpart for 77% of their formulas. On average, the truth values provide enough information to distinguish these two formulas. On the other hand, the truth value of a position of the trace. A formula that is undetermined under the operator whose operand is considered violated (satisfied) can contain the strong (weak) next-position of the pre-execution.

From LTL to rLTL: More Information; Same (Asymptotic) Cost

55.7% LTL-monitorable

10% rLTL-monitorable