



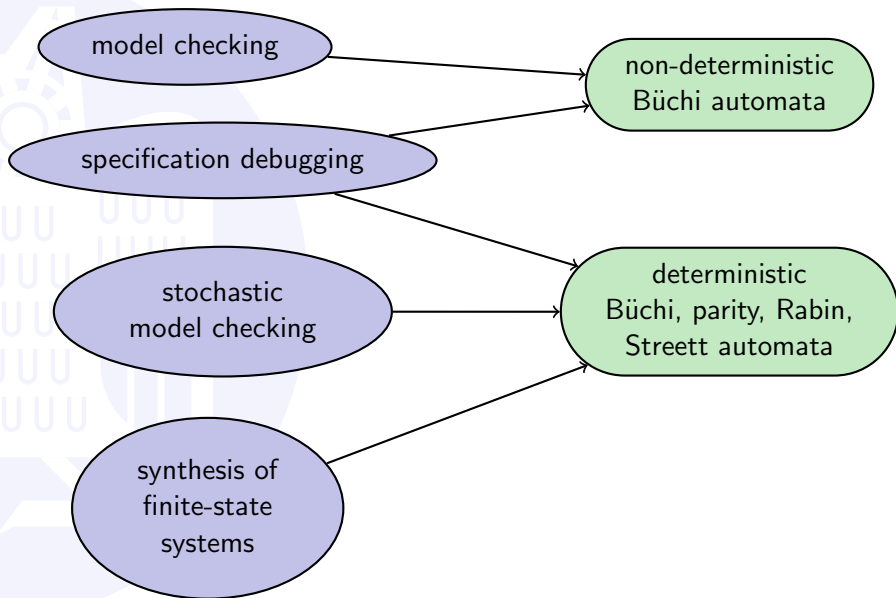
# Minimising Deterministic Büchi Automata Precisely using SAT Solving

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# Formal methods & $\omega$ -automata theory



# On $\omega$ -automata (1/2)

## Purpose

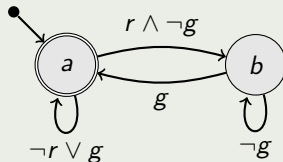
$\omega$ -automata accept or reject infinite words. In model checking, these normally correspond to runs of a system.

## Example system

A run of a gate-keeper with  
 $\Sigma = 2^{\{r,g\}}$ :

$$w = \begin{pmatrix} r = 1 \\ g = 0 \end{pmatrix} \begin{pmatrix} r = 0 \\ g = 1 \end{pmatrix} \begin{pmatrix} r = 0 \\ g = 0 \end{pmatrix} \dots$$

## Example Büchi automaton



## $\omega$ -automata types

**Branching types:** deterministic, non-deterministic, universal, alternating

**Acceptance conditions:** safety, Büchi, co-Büchi, parity, Rabin, Streett

# Deterministic Büchi automata (DBAs)

## Properties of DBAs

- Can be used for all purposes for which non-determinism is problematic (Stochastic model checking, synthesis, ...)
- Cannot express all  $\omega$ -regular properties (8/12 in our benchmark suite [EH00])
- Non-deterministic Büchi automata can be exponentially more succinct.
- DBAs are guaranteed to be not larger than equivalent deterministic parity or Rabin automata.

## DBAs in synthesis (from temporal logic formulas)

Heavily used in recent approaches:

- Generalized reactivity(1) synthesis [PPS06]
- Trigger property synthesis [KV10]

# Minimisation of $\omega$ -automata

## Complexity of the problems

Acc. cond. type	Non-deterministic	deterministic
Safety	PSPACE-comp.	in PTIME
Büchi	PSPACE-comp.	unknown
co-Büchi	PSPACE-comp.	unknown
Parity	PSPACE-comp.	unknown
Rabin	PSPACE-comp.	unknown
Streett	PSPACE-comp.	unknown

# Minimisation of $\omega$ -automata

## Complexity of the problems

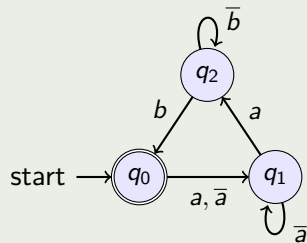
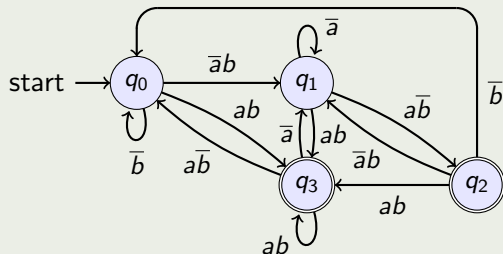
Acc. cond. type	Non-deterministic	deterministic
Safety	PSPACE-comp.	in PTIME
Büchi	PSPACE-comp.	in NP
co-Büchi	PSPACE-comp.	in NP
Parity	PSPACE-comp.	unknown
Rabin	PSPACE-comp.	unknown
Streett	PSPACE-comp.	unknown

## Main contribution of this paper

**For co-Büchi/Büchi automata, the problem is efficiently solvable by using a modern SAT solver!**

# Minimising DBA is difficult

Example for the LTL formula  $(GFa) \wedge (GFb)$



## Reduction to a decision problem

Basic idea: Repeatedly ask the question whether there is an equivalent DBA with **one less** state.

## Encoding the decision problem as a SAT instance

- **Guess** a smaller automaton
- **Guess** the reachable states in the product of the smaller automaton and the original automaton
- **Examine** if the product witnesses correctness
- Perform some **symmetry reduction**



# The Encoding - Part 1 (smaller automaton)

## Definition of DBA

Formal definition of a DBA:  $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$  with:

- $\delta : Q \times \Sigma \rightarrow Q$
- $F \subseteq Q$
- W.l.o.g. we assume that  $Q = \{0, 1, \dots, n-1\}$  and  $q_0 = 0$

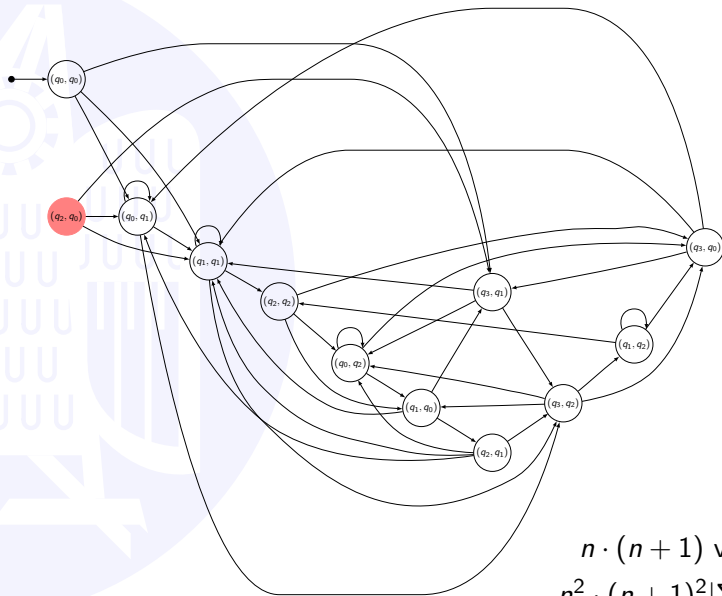
## SAT Encoding

We have the following number of variables in the SAT instance:

$$\underbrace{n}_{\text{for } F} + \underbrace{n^2 \cdot |\Sigma|}_{\text{for } \delta}$$

We need  $n \cdot |\Sigma|$  clauses for making sure that  $\delta(q, x)$  is defined for all  $x \in \Sigma$  and  $q \in Q$ .

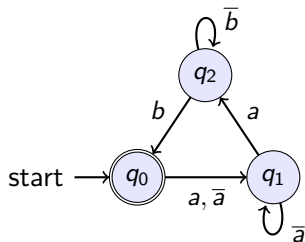
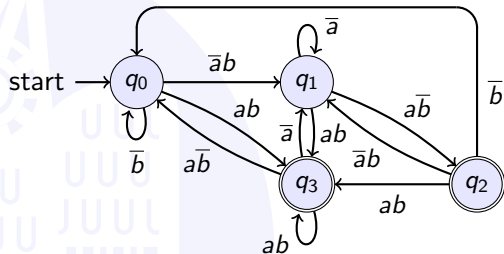
# The Encoding - Part 2 (reachable states in the product)



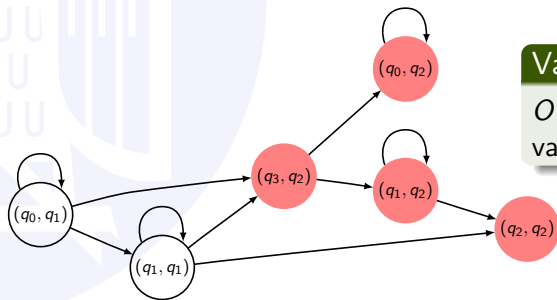
$n \cdot (n + 1)$  variables

$n^2 \cdot (n + 1)^2 |\Sigma|$  clauses

# The Encoding - Part 3 (examining the product)



**Var's and Clauses**  
 $O(n^4)$  many clauses and variables



## General properties

- Number of variables and clauses:  $O(n^4 \cdot |\Sigma|)$
- “Almost of Horn-type”: only step 1 leads to non-Horn clauses

## Symmetry reduction

In addition, we perform some (cheap) symmetry reduction (adds no variables or non-unit clauses).

## Properties

Example specifications:

- 8/12 LTL properties from [EH00] representable as DBAs
- 13 additional examples from verification & synthesis

## Tools

- Using `ltl2dstar` v.0.5.1 [KB06] in conjunction with `ltl2ba` v.1.1 [GO01] to obtain initial non-optimised deterministic Rabin automata equivalent to the input formulas.
- Converting these to Büchi automata whenever possible.
- Iteratively solving the resulting SAT instances with `picosat` v.913 [Bie08]

# Experiments - Results

LTL specification	# States		First instance		Total time
	From	To	# Vars	# Clauses	
$F(q \wedge X(pUr))$	3	3	94	697	0.01 s
$pUqUr \vee qUrUp \vee rUpUq$	3	3	112	699	0.01 s
$F(p \wedge X(q \wedge XFr))$	4	4	279	3785	0.01 s
$G(p \rightarrow qUr)$	5	3	852	13508	0.03 s
$pU(q \wedge X(rUs))$	5	5	780	26972	0.04 s
$F(p \wedge XF(q \wedge XF(r \wedge XFs)))$	5	5	780	26972	0.01 s
$GFp \wedge GFq \wedge GFr \wedge GFs \wedge GFu$	14	6	51467	13856026	63.03 s
$G(a \rightarrow Fb) \wedge Gc$	5	3	852	13508	0.03 s
$GF(a \rightarrow XXXb)$	7	2	3720	43457	0.08 s
$G(a \rightarrow Fb) \wedge G(\neg a \rightarrow F\neg b)$	8	4	5635	89511	0.14 s
$GF(a \leftrightarrow XXb)$	9	6	8760	168358	1.57 s
$G(a \rightarrow Fb) \wedge G(b \rightarrow Fc)$	10	5	14247	589925	0.98 s
$G(a \rightarrow XXXb)$	10	9	16623	295076	3.71 s
$G(a \rightarrow Fb) \wedge G(c \rightarrow Fd)$	15	6	72660	9926065	23.96 s
$GF(a \leftrightarrow XXXb)$	17	15	116912	4752970	3617.3 s*

## Contributions

- Minimising DBAs can efficiently be done by using a SAT solver
- Bringing the benefits of SAT solving to areas in which they are not used so far (stochastic model checking, synthesis)
- Actually, the technique is applicable to parity, Streett and Rabin automata as well
- Tool and hard SAT instances available at:  
<http://react.cs.uni-saarland.de/tools/dbaminimizer> and  
<http://react.cs.uni-saarland.de/~ehlers/benchmarks>

## A “wish list” for SAT solvers

- Allow streamed DIMACS input
- Delayed input instance simplification and symmetry breaking (during solving)

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