Generalised Rabin(1) synthesis[1]

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Overview of the talk

1. Synthesis of reactive systems - an overview
2. GR(1) - Generalised reactivity(1) synthesis
3. GRabin(1) - Generalised Rabin(1) synthesis
4. There's no room beyond GRabin(1)
5. What can we use GRabin(1) synthesis for?

Synthesis of reactive systems - overview

Problem description

Given ...

- a set of input atomic propositions AP_I,
- a set of output atomic propositions AP_O,
- a temporal logic formula ψ over $\mathsf{AP}_I \uplus \mathsf{AP}_O$

... does there exist a Mealy/Moore automaton reading AP_I and writing AP_O that satisfies ψ?

Properties of this problem

Church's problem is known to be 2EXPTIME-complete [PR89] for LTL specifications.

Incomplete overview of past synthesis approaches

Fast methods for subsets of LTL

- Safety synthesis
- Generalised reactivity(1) synthesis [PPS06]

Methods for full LTL

- Parity game based methods → Generalised parity game based methods [CHP07, SS09]
- Safraless synthesis [KV05] → Bounded synthesis [SF07]
- Rabin game based methods

GR(1) synthesis [PPS06] - Description

Form of the specification (f.t.p. of [KHB09])

$$(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \ldots \wedge g_m)$$

with:

- a set A of assumptions $a_1, \ldots, a_n$
- a set G of guarantees $g_1, \ldots, g_m$

such that:

- all elements in $A \cup G$ are deterministic Büchi automata
- all elements in $A \cup G$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

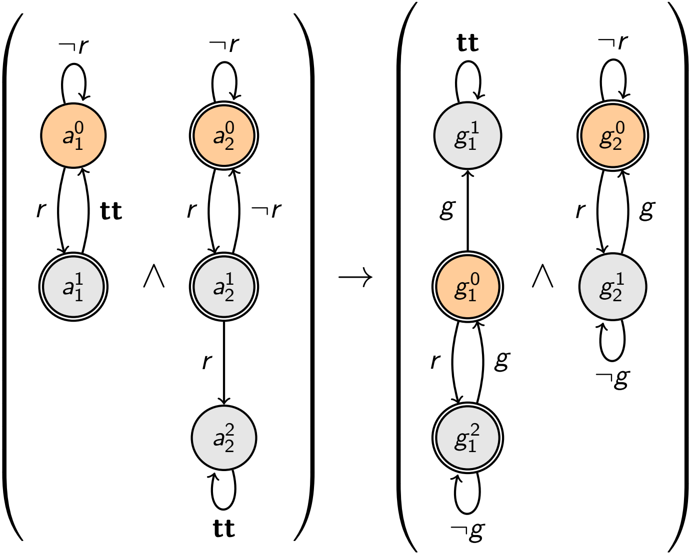
Explanation by example: a mutex

- Set of inputs: $\{r\}$ (a request)
- Set of outputs: $\{g\}$ (a grant)
- Assumptions:
  - $\mathsf{GF}\, r$
  - $\mathsf{G}(r \rightarrow \mathsf{X}\neg r)$
- Guarantees:
  - A grant is only issued after a request has been issued (since the last grant)
  - $\mathsf{GF}\, g$

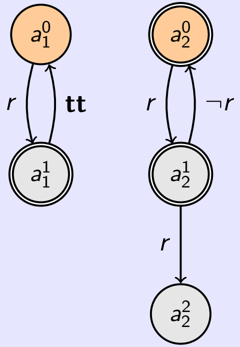
Basic idea of the algorithm

- Reduce the problem to solving a parity game with 3 colours : $0, 1, 2$
- The system player wins if the highest colour occurring infinitely often in the play is 0 or 2

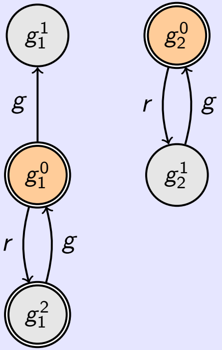
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



GR(1) synthesis - Algorithm (f.t.p. of [BCG$^+$10])

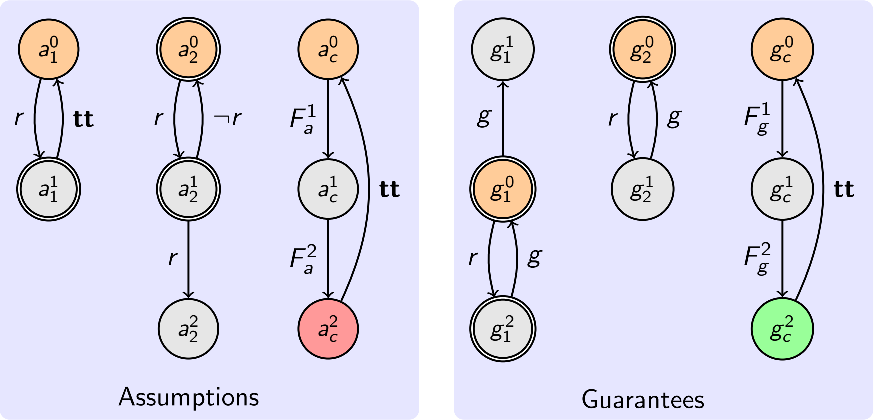


Assumptions



Guarantees

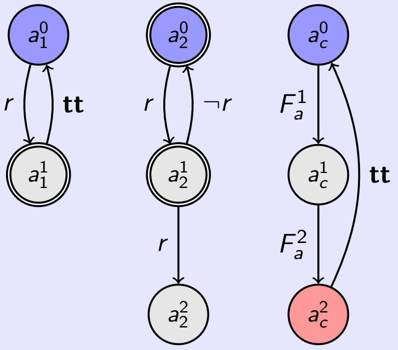
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



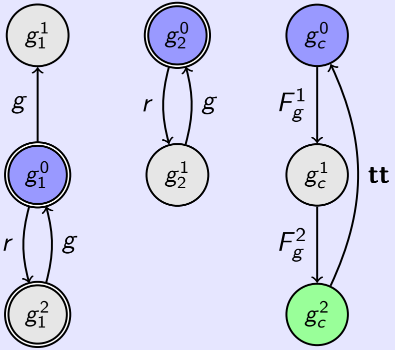
Main idea

Make a parity game over the product state space with 3 colours.

GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



Assumptions

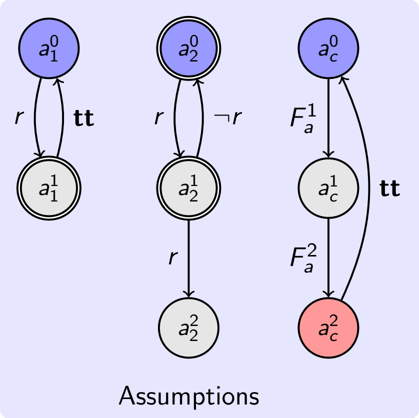


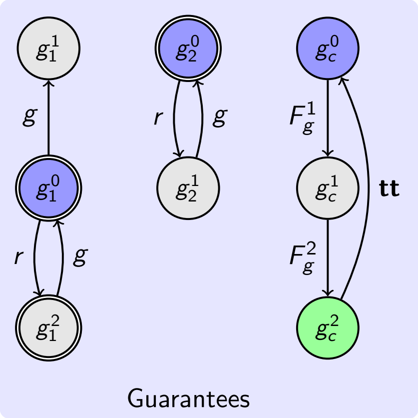
Guarantees

Example run

Request:			
Grant:			
Colour:			

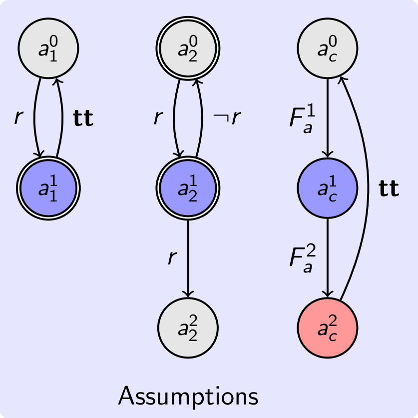
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

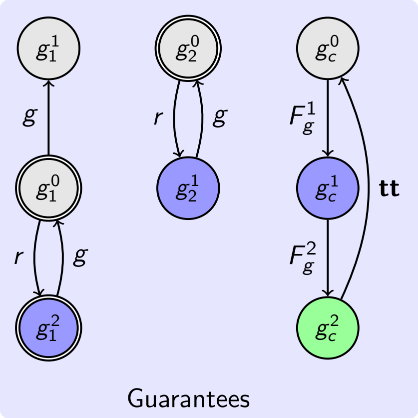




Example run

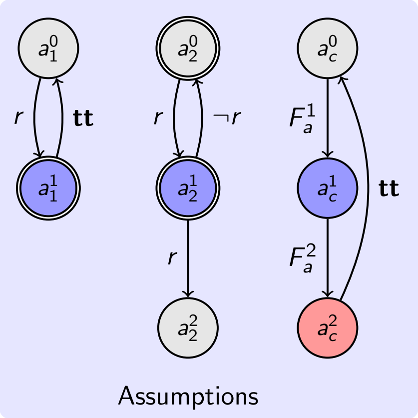
Request:	1		
Grant:			
Colour:			

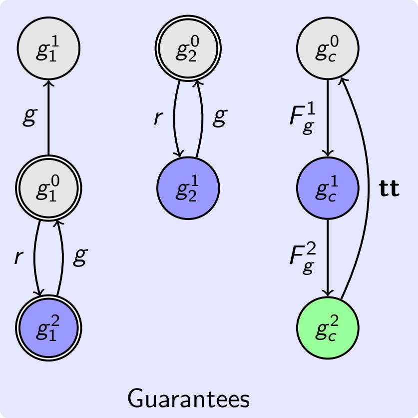




Example run

Request:	1		
Grant:	0		
Colour:	0		

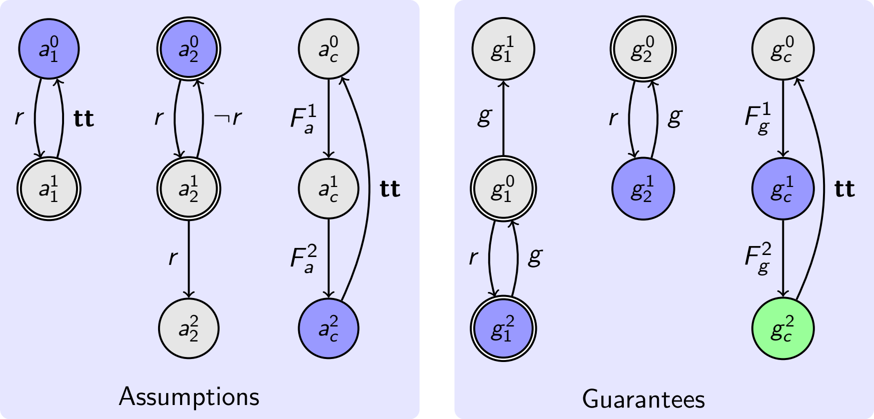




Example run

Request:	1	0
Grant:	0	
Colour:	0	

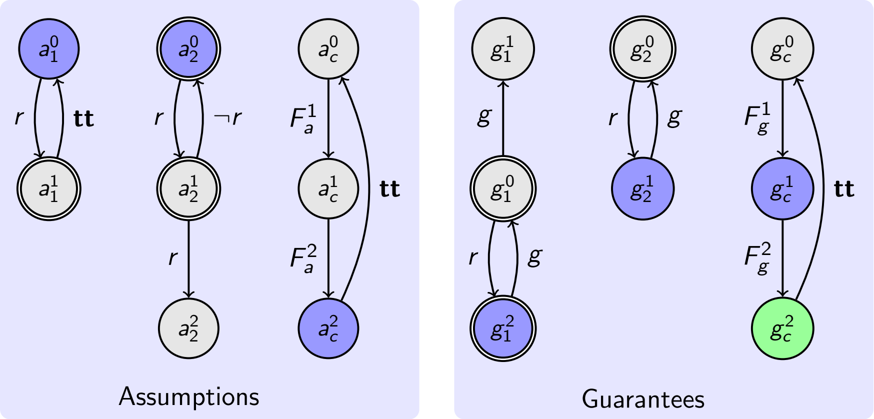
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



Example run

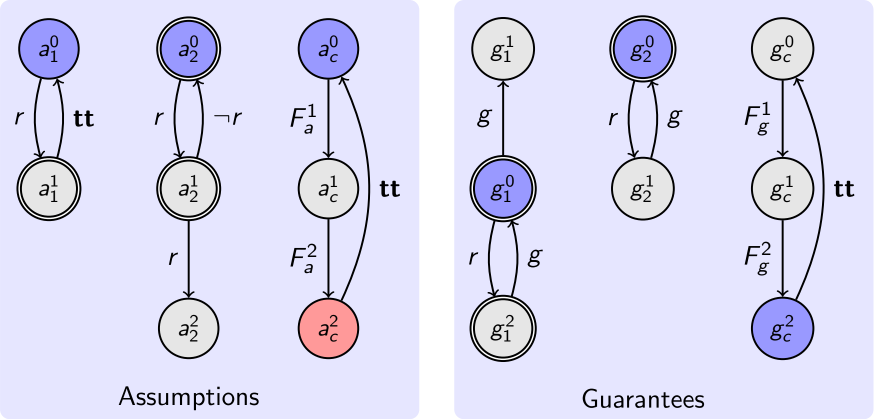
Request:	1	0
Grant:	0	0
Colour:	0	1

GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



Example run

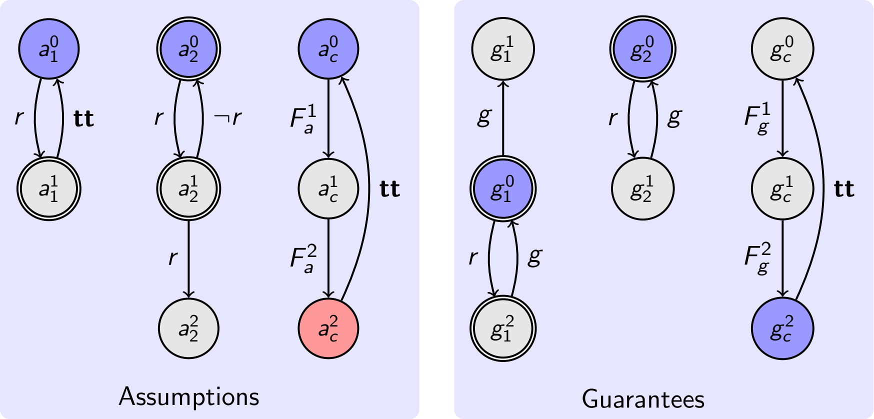
Request:	1	0	0
Grant:	0	0	
Colour:	0	1	



Example run

Request:	1	0	0
Grant:	0	0	1
Colour:	0	1	2

GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])



Example run

Request:	1	0	0
Grant:	0	0	1
Colour:	0	1	2

...

GR(1) synthesis - summary

The highest colour occurring infinitely often

- Colour 0 - Neither the assumptions nor the guarantees are fulfilled
- Colour 1 - The assumptions are fulfilled but not the guarantees
- Colour 2 - The guarantees are fulfilled

The nice properties of this approach

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- The parity game we obtain has a constant number of colours

$\Rightarrow$ Amenable to symbolic implementations, as confirmed by two case studies [BGJ$^+$07a, BGJ$^+$07b]

On extending GR(1)

Main question of this work

How far can we push the expressivity of GR(1) without losing its nice properties?:

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- We have a constant number of colours, independent of the number of assumptions and guarantees

Intuition on why we want this

- Many properties in practice cannot be expressed yet, e.g., FG(*ready*)

Answer to the question raised

A little bit further, but that's it then (assuming P$\neq$NP)

Generalised Rabin(1) synthesis

Form of the specification

$$(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \ldots \wedge g_m)$$

with:

- a set A of assumptions $a_1, \ldots, a_n$
- a set G of guarantees $g_1, \ldots, g_m$

such that:

- all elements in $A \cup G$ are deterministic **Rabin automata with one acceptance pair**
- all elements in $A \cup G$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

Rabin automata

Formal definition

A **one-pair** det. Rabin automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ with:

- a state set Q
- an alphabet Σ (here, $\Sigma = 2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$)
- a transition relation $Q \times \Sigma \rightarrow Q$
- an initial state $q_0 \in Q$
- an acceptance component $F = (F_1, F_2) \in 2^Q \times 2^Q$

We say that $\mathcal{A}$ accepts a run $\pi = \pi_0\pi_1\ldots$ if $\pi_0 = q_0$ and $\inf(\pi) \cap F_1 = \emptyset$ and $\inf(\pi) \cap F_2 \neq \emptyset$

A special property of one-pair Rabin automata

A run is accepted by a one-pair Rabin automaton $(Q, \Sigma, \delta, q_0, (F_1, F_2))$ iff it is accepted by the co-Büchi automaton $(Q, \Sigma, \delta, q_0, F_1)$ and the Büchi automaton $(Q, \Sigma, \delta, q_0, F_2)$.

Generalised Rabin(1) synthesis - simplified

Form of the specification

$$\begin{aligned} & (A_1 \wedge A_2 \wedge \ldots \wedge A_{|A|} \wedge B_1 \wedge B_2 \wedge \ldots \wedge B_{|B|}) \\ \rightarrow \quad & (C_1 \wedge C_2 \wedge \ldots \wedge C_{|C|} \wedge D_1 \wedge D_2 \wedge \ldots \wedge D_{|D|}) \end{aligned}$$

with:

- a set A of Büchi assumptions $A_1, \ldots, A_{|A|}$
- a set B of co-Büchi assumptions $B_1, \ldots, B_{|B|}$
- a set C of Büchi guarantees $C_1, \ldots, C_{|C|}$
- a set D of co-Büchi guarantees $D_1, \ldots, D_{|D|}$

such that:

- all elements in $A \cup C$ are deterministic Büchi automata
- all elements in $B \cup D$ are deterministic co-Büchi automata
- all elements in $A \cup B \cup C \cup D$ run over $2^{\mathsf{AP}_I \uplus \mathsf{AP}_O}$

GRabin(1) synthesis - solution idea

(Still flawed) reduction to a parity game

- State space is the product of the automata state spaces and the two Büchi assumption- and guarantee-checker automata A_C and G_C
- 5 colours:
  - Colour 4 - Some co-Büchi assumption is violated
  - Colour 3 - Some co-Büchi guarantee is violated
  - Colour 2 - The Büchi guarantee automata have "recently" all visited their accepting states
  - Colour 1 - The Büchi assumption automata have "recently" all visited their accepting states
  - Colour 0 - none of the above

GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play

	co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.	0	1	4	4
co-B. Gua. B. Gua.	2	2	4	4
B. Gua.	3	3	4	4
	3	3	4	4

GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play

	co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.	0	1	4	4
co-B. Gua. B. Gua.	2	2	4	4
B. Gua.	3	3	4	4
	3	3	4	4

A problem

Currently we lose on too many plays!

The solution

Add storage bit to the game tracking whether the counter automaton for A has "recently" completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

GRabin(1) synthesis - solution idea analysis

Maximal colour occurring inf. often on a play (fixed)

		co-B. Ass.	co-B. Ass. B. Ass.	B. Ass.	
co-B. Gua.		0	1	4	4
co-B. Gua.	B. Gua.	2	2	4	4
	B. Gua.	2	3	4	4
		0	3	4	4

A problem

Currently we lose on too many plays!

The solution

Add storage bit to the game tracking whether the counter automaton for *A* has "recently" completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

Can we generalise even further?

Answer: No!

Generalised Streett(1) synthesis cannot work in the way described:

- Game arena is the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees)
- Parity game with a constant number of colours

Reason

Generalised parity game solving is NP-hard for certain cases

Generalised parity games [CHP07]

The conjunctive version

- A game graph with k parity functions is given
- Player 0 needs to win for all of these functions at the same time

A hard case

For parity functions with the co-domain $\{0, 1, 2\}$, solving (the conjunctive version of) generalised parity games is NP-hard

A reduction of the hard case to generalised Streett(1) games

Convert such a game with k parity functions to the specification $(\mathbf{tt}) \rightarrow (G_1 \wedge \ldots \wedge G_k)$ for one-pair Streett guarantees G_i

The reduction (continued)

Examining the specification

$(\mathbf{tt}) \rightarrow (G_1 \wedge \ldots \wedge G_k)$ has a special property: $G_1, \ldots, G_k$ have the same transition structure.

Consequence

If the product game arena was the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees) and the game had a constant number of colours, we could solve an NP-hard problem in polynomial time.

An application: synthesis of robust systems

Example specification - A processing machine (base version)

$$\left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\\text{incoming}\end{array}\right) \wedge \mathsf{G}\left(\begin{array}{c}\text{no over-}\\\text{sized parts}\end{array}\right) \wedge \ldots\right)$$

$$\rightarrow \left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\\text{processed}\end{array}\right) \wedge \mathsf{G}\left(\begin{array}{c}\text{no}\\\text{jam}\end{array}\right) \wedge \ldots\right)$$

Example specification - A processing machine (robust)

$$\left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\\text{incoming}\end{array}\right) \wedge \mathsf{FG}\left(\begin{array}{c}\text{no over-}\\\text{sized parts}\end{array}\right) \wedge \ldots\right)$$

$$\rightarrow \left(\mathsf{GF}\left(\begin{array}{c}\text{part}\\\text{processed}\end{array}\right) \wedge \mathsf{FG}\left(\begin{array}{c}\text{no}\\\text{jam}\end{array}\right) \wedge \ldots\right)$$

Summary

Generalised Rabin(1) synthesis is ...

- a symbolically implementable synthesis method
- in some sense the best we can get (in this line of research)
- a practically relevant fragment of LTL

Important questions beyond the scope of this talk

- How to get from logic to Rabin(1) automata
- How to encode these automata into BDDs
- How to solve the resulting games efficiently

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