Generalised Rabin(1) synthesis

Rüdiger Ehlers

Saarland University, Reactive Systems Group

2nd Workshop on Games for Design, Verification and Synthesis – September 4, 2010

---

1 This work was supported by the German Research Foundation (DFG) within the program “Performance Guarantees for Computer Systems” and the Transregional Collaborative Research Center “Automatic Verification and Analysis of Complex Systems” (SFB/TR 14 AVACS).
Overview of the talk

1. Synthesis of reactive systems - an overview
2. GR(1) - Generalised reactivity(1) synthesis
3. GRabin(1) - Generalised Rabin(1) synthesis
4. There’s no room beyond GRabin(1)
5. What can we use GRabin(1) synthesis for?
Problem description

Given ...

- a set of input atomic propositions $AP_I$,
- a set of output atomic propositions $AP_O$,
- a temporal logic formula $\psi$ over $AP_I \cup AP_O$

... does there exist a Mealy/Moore automaton reading $AP_I$ and writing $AP_O$ that satisfies $\psi$?

Properties of this problem

Church’s problem is known to be $2\text{EXPTIME}$-complete [PR89] for LTL specifications.
Incomplete overview of past synthesis approaches

Fast methods for subsets of LTL

- Safety synthesis
- Generalised reactivity(1) synthesis [PPS06]

Methods for full LTL

- Parity game based methods
  - Generalised parity game based methods [CHP07, SS09]
- Safranless synthesis [KV05]
- Rabin game based methods
  - Bounded synthesis [SF07]
Form of the specification (f.t.p. of [KHB09])

\[(a_1 \land a_2 \land \ldots \land a_n) \rightarrow (g_1 \land g_2 \land \ldots \land g_m)\]

with:
- a set \(A\) of assumptions \(a_1, \ldots, a_n\)
- a set \(G\) of guarantees \(g_1, \ldots, g_m\)

such that:
- all elements in \(A \cup G\) are deterministic Büchi automata
- all elements in \(A \cup G\) run over \(2^{\text{AP}_I \cup \text{AP}_O}\)
Explanation by example: a mutex

- Set of inputs: \{r\} (a request)
- Set of outputs: \{g\} (a grant)
- Assumptions:
  - GF \(r\)
  - \(G(r \rightarrow X\neg r)\)
- Guarantees:
  - A grant is only issued after a request has been issued (since the last grant)
  - GF \(g\)

Basic idea of the algorithm

- Reduce the problem to solving a parity game with 3 colours: 0, 1, 2
- The system player wins if the highest colour occurring infinitely often in the play is 0 or 2
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

\[
\begin{pmatrix}
\neg r
\end{pmatrix}
\begin{pmatrix}
a_0^1
\end{pmatrix}
\begin{pmatrix}
r
\wedge
\end{pmatrix}
\begin{pmatrix}
a_1^1
\end{pmatrix}
\begin{pmatrix}
r
\wedge
\end{pmatrix}
\begin{pmatrix}
a_2^1
\end{pmatrix}
\begin{pmatrix}
r
\wedge
\end{pmatrix}
\begin{pmatrix}
a_2^2
\end{pmatrix}
\begin{pmatrix}
\rightarrow
\end{pmatrix}
\begin{pmatrix}
\neg r
\end{pmatrix}
\begin{pmatrix}
g_0^1
\end{pmatrix}
\begin{pmatrix}
r
\wedge
\end{pmatrix}
\begin{pmatrix}
g_1^0
\end{pmatrix}
\begin{pmatrix}
r
\wedge
\end{pmatrix}
\begin{pmatrix}
g_1^2
\end{pmatrix}
\begin{pmatrix}
\neg g
\end{pmatrix}
\begin{pmatrix}
\neg g
\end{pmatrix}
\begin{pmatrix}
\neg g
\end{pmatrix}
\begin{pmatrix}
\neg g
\end{pmatrix}
\end{pmatrix}
GR(1) synthesis - Algorithm (f.t.p. of [BCG\textsuperscript{+}10])

Assumptions

Guarantees
GR(1) synthesis - Algorithm (f.t.p. of [BCG\textsuperscript{+}10])

Main idea
Make a parity game over the product state space with 3 colours.
GR(1) synthesis - Algorithm (f.t.p. of [BCG^+10])

Example run

| Request: | |
|----------||
| Grant:   | |
| Colour:  | |
GR(1) synthesis - Algorithm (f.t.p. of [BCG⁺10])

Assumptions

Guarantees

Example run

<table>
<thead>
<tr>
<th>Request:</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant:</td>
<td></td>
</tr>
<tr>
<td>Colour:</td>
<td></td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG⁺10])

Assumptions

Guarantees

Example run

<table>
<thead>
<tr>
<th>Request</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant</td>
<td>0</td>
</tr>
<tr>
<td>Colour</td>
<td>0</td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG^10])

Assumptions

Guarantees

Example run

<table>
<thead>
<tr>
<th>Request:</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant:</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Colour:</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG⁺10])

Assumptions

Guarantees

Example run

<table>
<thead>
<tr>
<th>Request</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant:</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Colour:</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

Assumptions

Guarantees

Example run

<table>
<thead>
<tr>
<th>Request:</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant:</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Colour:</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

Assumptions

Example run

<table>
<thead>
<tr>
<th>Request:</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grant:</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Colour:</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
GR(1) synthesis - Algorithm (f.t.p. of [BCG+10])

**Example run**

| Request: | 1 | 0 | 0 |
| Grant:   | 0 | 0 | 1 | ... |
| Colour:  | 0 | 1 | 2 | ... |
GR(1) synthesis - summary

The highest colour occurring infinitely often

- Colour 0 - Neither the assumptions nor the guarantees are fulfilled
- Colour 1 - The assumptions are fulfilled but not the guarantees
- Colour 2 - The guarantees are fulfilled

The nice properties of this approach

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- The parity game we obtain has a constant number of colours
  \[\Rightarrow\text{Amenable to symbolic implementations, as confirmed by two case studies [BGJ}^{+07a, BGJ}^{+07b}\]
Main question of this work

How far can we push the expressivity of GR(1) without losing its nice properties?:

- The game arena for the parity game is the parallel composition of the Büchi automata and some polynomially sized control structure
- We have a constant number of colours, independent of the number of assumptions and guarantees

Intuition on why we want this

- Many properties in practice cannot be expressed yet, e.g., FG(ready)

Answer to the question raised

A little bit further, but that's it then (assuming P≠NP)
Form of the specification

\[(a_1 \land a_2 \land \ldots \land a_n) \rightarrow (g_1 \land g_2 \land \ldots \land g_m)\]

with:

- a set \(A\) of assumptions \(a_1, \ldots, a_n\)
- a set \(G\) of guarantees \(g_1, \ldots, g_m\)

such that:

- all elements in \(A \cup G\) are deterministic Rabin automata with one acceptance pair
- all elements in \(A \cup G\) run over \(2^{AP_I \cup AP_O}\)
Rabin automata

Formal definition

A **one-pair** det. Rabin automaton is a tuple \( A = (Q, \Sigma, \delta, q_0, F) \) with:

- a state set \( Q \)
- an alphabet \( \Sigma \) (here, \( \Sigma = 2^{AP_I \cup AP_O} \))
- a transition relation \( Q \times \Sigma \rightarrow Q \)
- an initial state \( q_0 \in Q \)
- an acceptance component \( F = (F_1, F_2) \in 2^Q \times 2^Q \)

We say that \( A \) accepts a run \( \pi = \pi_0\pi_1 \ldots \) if \( \pi_0 = q_0 \) and \( \text{inf}(\pi) \cap F_1 = \emptyset \) and \( \text{inf}(\pi) \cap F_2 \neq \emptyset \)

A special property of one-pair Rabin automata

A run is accepted by a one-pair Rabin automaton \( (Q, \Sigma, \delta, q_0, (F_1, F_2)) \) iff it is accepted by the co-Büchi automaton \( (Q, \Sigma, \delta, q_0, F_1) \) and the Büchi automaton \( (Q, \Sigma, \delta, q_0, F_2) \).
Form of the specification

\[(A_1 \land A_2 \land \ldots \land A_{|A|} \land B_1 \land B_2 \land \ldots \land B_{|B|}) \rightarrow (C_1 \land C_2 \land \ldots \land C_{|C|} \land D_1 \land D_2 \land \ldots \land D_{|D|})\]

with:
- a set \( A \) of Büchi assumptions \( A_1, \ldots, A_{|A|} \)
- a set \( B \) of co-Büchi assumptions \( B_1, \ldots, B_{|B|} \)
- a set \( C \) of Büchi guarantees \( C_1, \ldots, C_{|C|} \)
- a set \( D \) of co-Büchi guarantees \( D_1, \ldots, D_{|D|} \)

such that:
- all elements in \( A \cup C \) are deterministic Büchi automata
- all elements in \( B \cup D \) are deterministic co-Büchi automata
- all elements in \( A \cup B \cup C \cup D \) run over \( 2^{AP_I \cup AP_O} \)
GRabin(1) synthesis - solution idea

(Still flawed) reduction to a parity game

- State space is the product of the automata state spaces and the two Büchi assumption- and guarantee-checker automata $A_C$ and $G_C$
- 5 colours:
  - Colour 4 - Some co-Büchi assumption is violated
  - Colour 3 - Some co-Büchi guarantee is violated
  - Colour 2 - The Büchi guarantee automata have “recently” all visited their accepting states
  - Colour 1 - The Büchi assumption automata have “recently” all visited their accepting states
  - Colour 0 - none of the above
**Maximal colour occurring inf. often on a play**

<table>
<thead>
<tr>
<th></th>
<th>co-B. Ass.</th>
<th>co-B. Ass.</th>
<th>B. Ass.</th>
<th>B. Ass.</th>
</tr>
</thead>
<tbody>
<tr>
<td>co-B. Gua.</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>co-B. Gua.</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B. Gua.</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B. Gua.</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Currently we lose on too many plays! The solution: Add storage bit to the game tracking whether the counter automaton for $A_1$ has "recently" completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).
### A problem

Currently we lose on too many plays!

### The solution

Add storage bit to the game tracking whether the counter automaton for $A$ has “recently” completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

#### Maximal colour occurring inf. often on a play

<table>
<thead>
<tr>
<th></th>
<th>co-B. Ass.</th>
<th>co-B. Ass.</th>
<th>B. Ass.</th>
<th>B. Ass.</th>
</tr>
</thead>
<tbody>
<tr>
<td>co-B. Gua.</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>co-B. Gua. B. Gua.</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B. Gua.</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
### A problem
Currently we lose on too many plays!

### The solution
Add storage bit to the game tracking whether the counter automaton for $A$ has “recently” completed a cycle. Only use colour 3 if this was the case (and reset the bit in this case).

<table>
<thead>
<tr>
<th>Maximal colour occurring inf. often on a play (fixed)</th>
<th>co-B. Ass.</th>
<th>co-B. Ass. B. Ass.</th>
<th>B. Ass.</th>
<th>B. Ass.</th>
</tr>
</thead>
<tbody>
<tr>
<td>co-B. Gua.</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>co-B. Gua. B. Gua.</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>B. Gua.</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
Can we generalise even further?

Answer: No!

Generalised Streett(1) synthesis cannot work in the way described:

- Game arena is the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees)
- Parity game with a constant number of colours

Reason

Generalised parity game solving is NP-hard for certain cases
Generalised parity games [CHP07]

The conjunctive version

- A game graph with \( k \) parity functions is given
- Player 0 needs to win for all of these functions at the same time

A hard case

For parity functions with the co-domain \( \{0, 1, 2\} \), solving (the conjunctive version of) generalised parity games is NP-hard

A reduction of the hard case to generalised Streett(1) games

Convert such a game with \( k \) parity functions to the specification \( (tt) \rightarrow (G_1 \land \ldots \land G_k) \) for one-pair Streett guarantees \( G_i \).
The reduction (continued)

Examining the specification

\[(\texttt{tt}) \rightarrow (G_1 \land \ldots \land G_k)\] has a special property: \(G_1, \ldots, G_k\) have the same transition structure.

Consequence

If the product game arena was the product of the individual automata and a polynomially sized control structure (in the number of assumptions and guarantees) and the game had a constant number of colours, we could solve an NP-hard problem in polynomial time.
An application: synthesis of robust systems

Example specification - A processing machine (base version)

\[
(GF(part\ incoming) \land G(no\ oversized\ parts) \land \ldots) \\
\Rightarrow (GF(part\ processed) \land G(no\ jam) \land \ldots)
\]

Example specification - A processing machine (robust)

\[
(GF(part\ incoming) \land FG(no\ oversized\ parts) \land \ldots) \\
\Rightarrow (GF(part\ processed) \land FG(no\ jam) \land \ldots)
\]
Generalised Rabin(1) synthesis is . . .

- a symbolically implementable synthesis method
- in some sense the best we can get (in this line of research)
- a practically relevant fragment of LTL

Important questions beyond the scope of this talk

- How to get from logic to Rabin(1) automata
- How to encode these automata into BDDs
- How to solve the resulting games efficiently
Roderick Bloem, Krishnendu Chatterjee, Karin Greimel, Thomas A. Henzinger, and Barbara Jobstmann.  
Robustness in the presence of liveness.  

Interactive presentation: Automatic hardware synthesis from specifications: a case study.  

Specify, compile, run: Hardware from PSL.  

Krishnendu Chatterjee, Thomas A. Henzinger, and Nir Piterman.  
Generalized parity games.  

Robert Könighofer, Georg Hofferek, and Roderick Bloem.  
Debugging formal specifications using simple counterstrategies.  

Orna Kupferman and Moshe Y. Vardi.  
Safraless decision procedures.  

Nir Piterman, Amir Pnueli, and Yaniv Sa’ar.  
Synthesis of reactive(1) designs.  
Amir Pnueli and Roni Rosner.
On the synthesis of an asynchronous reactive module.

Sven Schewe and Bernd Finkbeiner.
Bounded synthesis.

Saqib Sohail and Fabio Somenzi.
Safety first: A two-stage algorithm for LTL games.