Verification

Lecture 9

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Plan for today

- Regular properties
  - Finite automata
  - Checking regular safety properties
  - Büchi automata

Review: ω-regular expressions

1. $\underline{\varnothing}$ and $\underline{\varepsilon}$ are regular expressions over Σ
2. if $A \in \Sigma$ then $\underline{A}$ is a regular expression over Σ
3. if E, E_1 and E_2 are regular expressions over Σ then so are $E_1 + E_2$, $E_1.E_2$ and E^*

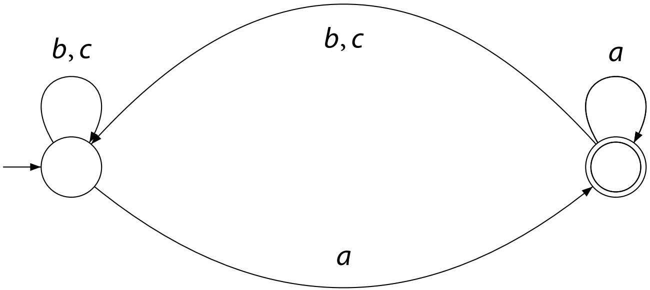
E^+ is an abbreviation for the regular expression $E.E^*$

An ω-regular expression G over the alphabet Σ has the form:

$$G = E_1.F_1^{\omega} + \ldots + E_n.F_n^{\omega} \quad \text{for } n > 0$$

where E_i, F_i are regular expressions over Σ such that $\varepsilon \notin \mathcal{L}(F_i)$, for all $0 < i \leq n$

Review: Büchi automata



NBA are more expressive than DBA

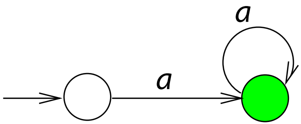
NFA and DFA are equally expressive but NBA and DBA are not!

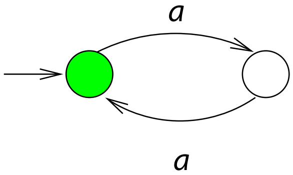
There is no DBA that accepts $\mathcal{L}_\omega((A + B)^* B^\omega)$

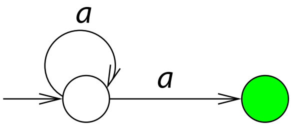
Proof

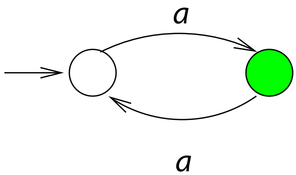
- Assume that $L = \mathcal{L}((A+B)^*B^\omega)$ is recognized by the deterministic Büchi automaton $\mathcal{A}$.
- Since $b^\omega \in L$, there is a run
  $r_0 = s_{0,0}s_{0,1}s_{0,2}, \ldots$
  with $s_{0,n_0} \in F$ for some $n_0 \in \mathbb{N}$.
- Similarly, $b^{n_0}ab^\omega \in L$ and there must be a run
  $r_1 = s_{0,0}s_{0,1}s_{0,2} \ldots s_{0,n_0}s_1s_{1,0}s_{1,1}s_{1,2} \ldots$
  with $s_{1,n_1} \in F$
- Repeating this argument, there is a word
  $b^{n_0}ab^{n_1}ab^{n_2}a \ldots$
  accepted by $\mathcal{A}$.
- This contradicts $L = \mathcal{L}_\omega(\mathcal{A})$.

NBA versus NFA









finite equivalence
$\not\Rightarrow$ ω-equivalence

$\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$,
but $\mathcal{L}_\omega(\mathcal{A}) \neq \mathcal{L}_\omega(\mathcal{A}')$

ω-equivalence
$\not\Rightarrow$ finite equivalence

$\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{A}')$,
but $\mathcal{L}(\mathcal{A}) \neq \mathcal{L}(\mathcal{A}')$

NBA and ω-regular languages

The class of languages accepted by NBA
agrees with the class of ω-regular languages

(1) any ω-regular language is recognized by an NBA

(2) for any NBA $\mathcal{A}$, the language $\mathcal{L}_\omega(\mathcal{A})$ is ω-regular

For any ω-regular language there is an NBA

- How to construct an NBA for the ω-regular expression:

$$G = E_1.F_1^{\omega} + \ldots + E_n.F_n^{\omega} \ ?$$

where E_i and F_i are regular expressions over alphabet Σ; $\varepsilon \notin F_i$

- Rely on operations for NBA that mimic operations on ω-regular expressions:
  (1) for NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ there is an NBA accepting $\mathcal{L}_\omega(\mathcal{A}_1) \cup \mathcal{L}_\omega(\mathcal{A}_2)$
  (2) for any regular language $\mathcal{L}$ with $\varepsilon \notin \mathcal{L}$ there is an NBA accepting $\mathcal{L}^\omega$
  (3) for regular language $\mathcal{L}$ and NBA $\mathcal{A}'$ there is an NBA accepting $\mathcal{L}.\mathcal{L}_\omega(\mathcal{A}')$

Union of NBA

For NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ (both over the alphabet Σ)

there exists an NBA $\mathcal{A}$ such that:

$\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{A}_1) \cup \mathcal{L}_\omega(\mathcal{A}_2)$ and $|\mathcal{A}| = \mathcal{O}(|\mathcal{A}_1| + |\mathcal{A}_2|)$

Proof on blackboard!

ω-operator for NFA

For each NFA $\mathcal{A}$ with $\varepsilon \notin \mathcal{L}(\mathcal{A})$ there exists an NBA $\mathcal{A}'$ such that:

$$\mathcal{L}_\omega(\mathcal{A}') = \mathcal{L}(\mathcal{A})^\omega \quad \text{and} \quad |\mathcal{A}'| = \mathcal{O}(|\mathcal{A}|)$$

Proof on blackboard!