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Reactive Systems Group
Saarland University
Due: Thursday, 5th February 2009
before the lecture, i.e., 2:10pm

Embedded Systems 08/09 – Problem Set 10

Problem 1 - Real-Time Calculus (40 pts.)

Figure 1 shows an MPEG-2 decoder application that is partitioned and mapped onto two processors P1 and P2. The (coded) input stream enters this system and it is stored in the input buffer B. The macroblocks in B are first processed by P1 and the corresponding partially decoded macroblocks are stored in the buffer B′ before being processed by P2. The resulting stream of fully decoded macroblocks is written into a playout buffer B″ prior to transmission by the output video device.

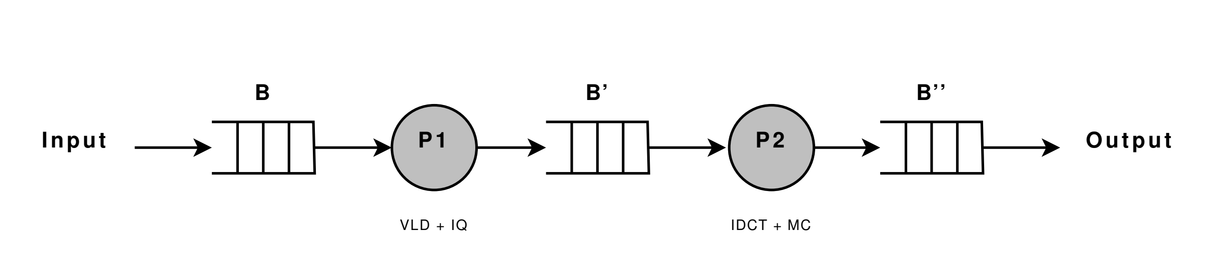

Figure 1: An MPEG-2 decoder with a Variable Length Decoding (VLD), an Inverse Quantization (IQ), an Inverse Discrete Cosine Transform (IDCT), and a Motion Compensation (MC) unit.

In the above system, the coded input event stream arrives at a constant speed of 128 kbps.[1] Processor P1 can either run with a slow speed of 100 MHz or with a high speed of 400 MHz.[2] Timing analysis shows a BCET of 4 cycles per bit and a WCET of 10 cycles per bit.

Please perform the following tasks:

1. Give P1's incoming arrival and service functions (α_1^l, α_1^u) and (β_1^l, β_1^u), respectively. (10 pts.)
2. Compute P1's outgoing (P2's incoming) arrival functions (α_2^l, α_2^u). (20 pts.)
3. Assuming the same processor type for P2, at which speed f_2 has P2 to run such that no buffer overflow at B′ can occur. (10 pts.)

[1] i.e., $128 \cdot 10^3$ bits arrive every second.
[2] i.e., P1 executes $100 \cdot 10^6$ or $400 \cdot 10^6$ cycles per second, respectively.

Problem 2 (Static Redundancy) (30 pts.)

Figure 2 shows the *triple modular redundancy* arrangement described during the lecture. It uses three identical parallel working modules, and works correcly as far as at least two of the three modules are intact.

1. Assume that all three modules have the same reliability $R(t)$. Also assume that the voter is absolutely reliable. Prove that under these assumptions the reliability of the TMR arrangement is
$$R_{TMR}(t) = 3R^2(t) - 2R^3(t).$$
(10 pts.)

2. Is the realiability of the TMR arrangement always higher than that of a single module? Justify your answer with a concrete example. (10 pts.)

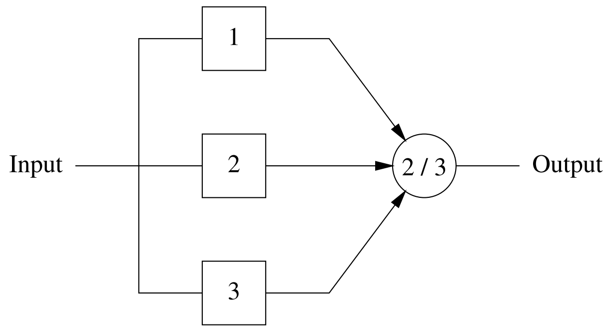

Figure 2: TMR arrangement.

3. When a module has a constant failure rate λ then its reliability falls exponentially according to the law $R(t) = e^{-\lambda t}$. At what point of time is the deployment of the TMR versus one single module no more justified? (10 pts.)

Problem 3 - Reliability Analysis (30 pts.)

Figure 3 shows the reliability block diagram of a system. The numbers in the blocks give the reliability of the pertaining system component.

1. Compute the reliability of the whole system. Show the computation and all partial results. (10 pts.)

2. A MINIMAL CUT SET is a minimal set of components whose collective failure causes the failure of the whole system. Minimal means that the failure of all components of a proper subset of the cut set does *not* cause the failure of the whole system.

   Determine all the minimal cut sets of the given system. Give the minimal cut sets by listing the identifying digits of the components belonging to the cut sets. (10 pts.)

3. Given all minimal cut sets of a system allows one to determine a lower bound for the reliability of the system. One has:
$$R(t) > 1 - \sum_j \prod_i \left[1 - R_i(t)\right] ,$$

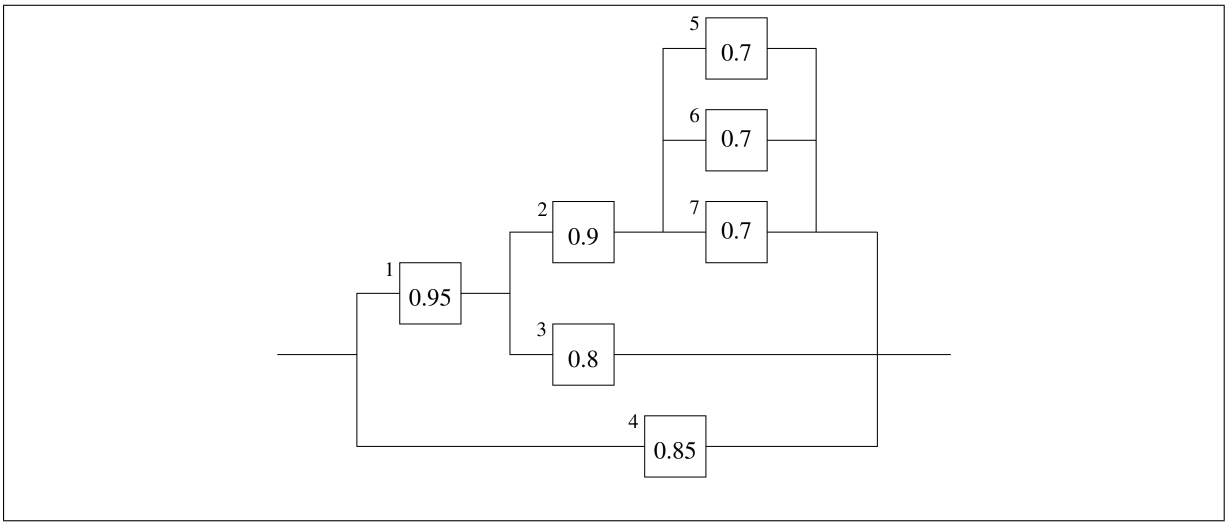

Figure 3: Reliability diagram.

where j ranges over all cut sets, and i ranges over all componenents of the jth cut set. Compute the bound and compare it with the exact bound computed in subproblem 1. (10 pts.)