Prof. Bernd Finkbeiner, Ph.D.
Dipl.-Inf. Rüdiger Ehlers
Dipl.-Inf. Hans-Jörg Peter
Michael Gerke, B.Sc.

Reactive Systems Group
Saarland University
Due: Thursday, 29th January 2009
before the lecture, i.e., 2:10pm

Embedded Systems 08/09 – Problem Set 9

Problem 1 - Abstract caches (30 pts.)

An abstract cache can be represented as a partial function $f : \mathbb{N}_0 \rightharpoonup 2^V$ that maps age values (positive integers) to a subset of a finite object set V. Using this representation having the object set $V = \{a, b, c, d, e\}$, the abstract cache with 4 entries

$\{a\}$
$\{b, c\}$
$\{d\}$
$\{e\}$

can be represented as the partial function f with

$$\begin{aligned} f(0) &= \{a\}, \\ f(1) &= \{b, c\}, \\ f(2) &= \{d\}, \quad \text{and} \\ f(3) &= \{e\}. \end{aligned}$$

Assuming an LRU caching strategy, your task is to prove the following claims:

1. For an arbitrary abstract *must* cache f and an object $x \in V$, let f' be the resulting abstract *must* cache after accessing x in f. Then the following holds true:

$$\forall i. \left(x \in f(i) \implies f(i) \setminus \{x\} \subseteq f'(i)\right)$$

   (15 pts.)

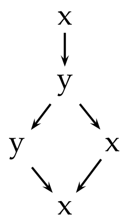
2. For an arbitrary abstract *may* cache f with n entries and an object $x \in V$, let f' be the resulting abstract *may* cache after accessing x in f. Then the following holds true:

$$f'(0) = \{x\} \quad \wedge \quad \forall i < n - 1. \left(f'(i+1) = f(i) \setminus \{x\}\right)$$

   (15 pts.)

Problem 2 - Cache analysis and predictability (30 pts.)

FIFO is an alternative cache-replacement policy. In case of a cache-miss, the oldest cache entry is removed. Consider the following sequence of accesses:



1. Assume an LRU-cache with 2 entries. Apply a must-cache and a may-cache analysis of the program. Can you predict the exact cache state at the last access to x? (10 pts.)
2. Assume a FIFO-cache with 2 entries and an empty cache at the beginning. Can you predict a cache-hit or a cache-miss at the last access to x? (10 pts.)
3. Repeat (2) with an unknown cache-state at the beginning of the program. What can you say about the predictability of an LRU-cache compared to a FIFO-cache? (10 pts.)

Problem 3 - Abstract Interpretation (20 pts.)

For a positive integer $n \in \mathbb{N}_0$ consider the lattices

$$\begin{aligned} C &= (2^{\mathbb{N}_0}, \subseteq, \top_C, \bot_C) \quad \text{and} \\ A &= (2^{\{0,\ldots,n-1\}}, \subseteq, \top_A, \bot_A), \end{aligned}$$

as well as the two transfer functions

$$\begin{aligned} \alpha_n &: 2^{\mathbb{N}_0} \longrightarrow 2^{\{0,\ldots,n-1\}} \quad \text{and} \\ \gamma_n &: 2^{\{0,\ldots,n-1\}} \longrightarrow 2^{\mathbb{N}_0} \end{aligned}$$

which are defined as

$$\begin{aligned} \alpha_n(c) &= \{x' \in \mathbb{N}_0 \mid \exists x \in c.\ x' \equiv x \bmod n\}, \\ \gamma_n(a) &= \{x' \in \mathbb{N}_0 \mid \exists x \in a\, \exists k \in \mathbb{N}_0.\ x' = kn + x\}, \end{aligned}$$

for any $c \in 2^{\mathbb{N}_0}$ and $a \in 2^{\{0,\ldots,n-1\}}$.

1. Give suitable top and bottom elements $\top_C$, $\bot_C$, $\top_A$, and $\bot_A$. (5 pts.)
2. Prove that (α_n, γ_n) is a Galois connection. (15 pts.)

Problem 4 - Path analysis (20 pts.)

Consider the following (pseudo code) program and its corresponding local WCET:

```
if (a) then
  if (b) then
    c ; d
  elseif (e) then
    f
  else
    g ; h
else
  i ; j
```

Statement	WCET
a	10
b	2
c	4
d	3
e	1
f	8
g	6
h	2
i	7
j	3

1. Give the control flow graph (in a graphical notation). (5 pts.)
2. Give an integer linear program for maximizing the overall WCET. (10 pts.)
3. Compute the overall WCET and its corresponding path. (5 pts.)